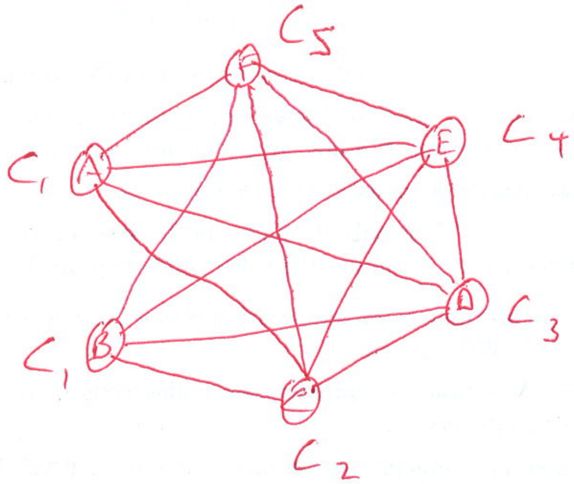
1. (a) ${}^6C_2 = 15$, so only one possible edge is missing. Let it be between nodes/vertices A and B. Then, any subset with 5 nodes, which doesn't include both A and B, will form a clique. So the graph has 2 5-cliques (one excluding A, one excluding B), and the answer is <u>5</u>.

(b) 5 is a lower bound on the min. no. of colours needed. So it must be 5 or 6.

(c) The nodes A and B (as per part (a)) can certainly be given the same colour, because there is no edge between them, and every other node will have a different colour. So A and B having the same colour will not cause any conflicts.



$c_1, \dots c_5$ denote the colours

Hence the min. no. of colours is definitely <u>5</u>.

2. (a) Defn. I : Yes, choose $\underline{c=1,\ n_0=101}$.

Defn. II : Yes, choose $\underline{c=1}$, infinite sequence of $\underline{n=\{101, 102, 103, \ldots\}}$.

(b) Assume $T(n)$ is $\Omega(f(n))$. So:

$\exists\, c>0,\ n_0$ s.t. $T(n) \geq c f(n)\ \forall n \geq n_0$

$\Rightarrow \underbrace{\tfrac{1}{c}}_{\triangleq c'\ (>0)} T(n) \geq f(n)\ \forall n \geq n_0$

$\Rightarrow f(n) \leq c' T(n)\ \forall n \geq n_0$

$\Rightarrow f(n)$ is $O(T(n))$, with same n_0, and $c' = \frac{1}{c}$, used to satisfy the definition.

Now assume $f(n)$ is $O(T(n))$. So:

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$\Rightarrow T(n) \geq c' f(n)\ \forall n \geq n_0$

$\Rightarrow T(n)$ is $\Omega(f(n))$, with same n_0, and $c' = \frac{1}{c}$, used to satisfy the definition.

Hence proved in both directions:

$\underline{T(n) \text{ is } \Omega(f(n)) \iff f(n) \text{ is } O(T(n))}$

(c) Is NOT true for Defn. II. Counter example:

$$T(n) = \begin{cases} (n+1)^2 & \text{if } n \text{ odd} \\ n & \text{if } n \text{ even} \end{cases}$$

$T(n)$ is $\Omega(n^2)$: choose $c=1$, $n = \{1, 3, 5, \ldots\}$

But n^2 is not $O(T(n))$: we cannot find an n_0, c s.t. $n^2 \le cT(n)$ $\forall n \ge n_0$, as for even n, the ratio n^2/n grows arbitrarily large with n, and so no finite c will work $\forall n \ge n_0$.

3. (a)

```
struct node {
    int element;
    struct node* next;
}
```

(b)

```
struct node* END (struct node* L) {
    while (L -> next != NULL)
        L = L -> next;

    return L;
}
```