ELL781: Software Fundamentals for Computer Technology

Minor Test II, Form: $\boxed{\text{A}}$ (**please write this Form ID on the cover page of your answer script**)
Maximum marks: 15

Section 1. Short Answer Questions

NB: Please read all questions carefully. There may be subtle differences between what the question is asking for and the context in which things have been discussed during the lectures, and these have to be taken into account whilst answering.

1. Consider the Towers of Hanoi problem discussed in class:

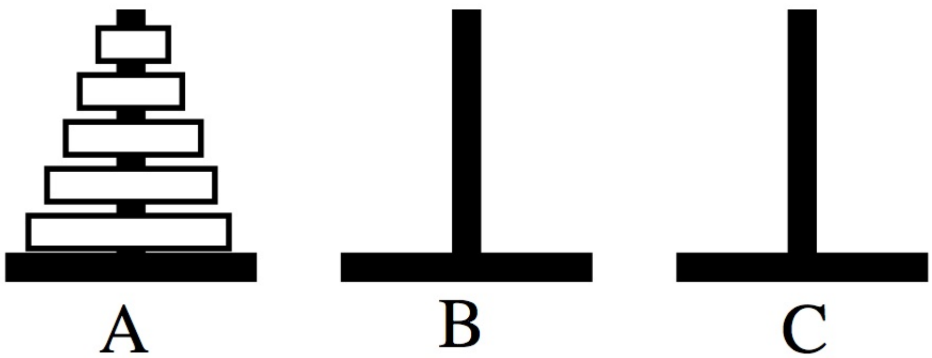

Figure 1: Towers of Hanoi (Copyright J. A. Storer, Brandeis University)

Recall, all the disks are to be moved from Peg A to Peg B, such that a larger disk is never placed on top of a smaller one. Let us define $T(n)$ as the total number of disk moves required to accomplish this, where n is the number of disks. Obtain a simple closed-form expression for $T(n)$. (You may use any approach you wish, but make sure to write out your working and reasoning fully and clearly.) **[4]**

2. Suppose I initialise a new `PRIORITYQUEUE Q`, which is meant to store positive integers such that bigger integers have higher priority. I call the `INSERT` and `DELETEMAX` operations on it in the following sequence:

- `INSERT(4,Q)`
- `INSERT(7,Q)`
- `INSERT(6,Q)`
- `INSERT(12,Q)`
- `DELETEMAX(Q)`
- `INSERT(9,Q)`
- `INSERT(8,Q)`
- `DELETEMAX(Q)`

(a) Assume that the `PRIORITYQUEUE` is implemented using a partially ordered tree. Show what the tree will be after each of the above function calls. Also show the output of each function call, if any. **[2]**

(b) Assume further that the partially ordered tree is stored in memory as a heap. Show the heap representation of the *final tree* only, after all of the above function calls. **[1]**

3. Recall the function `MEMBER(x,A)`, which checks for the membership of element `x` in a binary search tree (BST) `A`. In class, we wrote down a recursive version of this function. Now write the pseudocode for a *non-recursive* implementation of it. (You may assume the elements to be integers, and use the same pointer-based BST implementation which was introduced in class; please make sure all your notation / variable names are completely clear.) **[3]**

4. Consider the following cost matrix for a directed graph (infinite cost denotes no direct edge):

	1	**2**	**3**
1	2	8	3
2	2	∞	7
3	∞	2	∞

(a) Draw the corresponding directed graph, also showing the edge weights. **[0.5]**

(b) Run the Floyd-Warshall algorithm on this graph. Show the full A and P matrices at each iteration, starting with A_0 and P as a matrix of all zeroes. **[3.5]**

(c) Recall the recursive function $path(i, j)$, which uses the final P matrix to print out the shortest path between any pair of nodes (i, j). Show the full recursion tree and output resulting from the call $path(2, 3)$. [**1**]

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Minor Test II, Form: $\boxed{\text{B}}$ (**please write this Form ID on the cover page of your answer script**)

Maximum marks: 15

Section 1. Short Answer Questions

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1. Consider the Towers of Hanoi problem discussed in class:

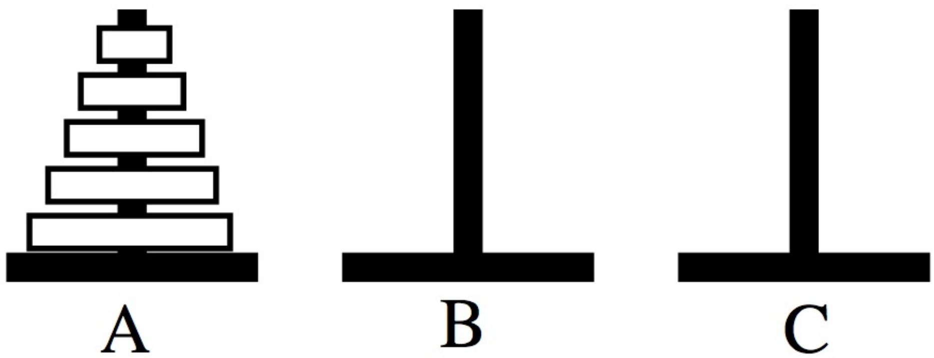

Figure 2: Towers of Hanoi (Copyright J. A. Storer, Brandeis University)

Recall, all the disks are to be moved from Peg A to Peg B, such that a larger disk is never placed on top of a smaller one. Let us define $T(n)$ as the total number of disk moves required to accomplish this, where n is the number of disks. Obtain a simple closed-form expression for $T(n)$. (You may use any approach you wish, but make sure to write out your working and reasoning fully and clearly.) **[4]**

2. Suppose I initialise a new `PRIORITYQUEUE Q`, which is meant to store positive integers such that bigger integers have higher priority. I call the `INSERT` and `DELETEMAX` operations on it in the following sequence:

- `INSERT(5,Q)`
- `INSERT(6,Q)`
- `INSERT(3,Q)`
- `INSERT(2,Q)`
- `DELETEMAX(Q)`
- `INSERT(4,Q)`
- `INSERT(9,Q)`
- `DELETEMAX(Q)`

(a) Assume that the `PRIORITYQUEUE` is implemented using a partially ordered tree. Show what the tree will be after each of the above function calls. Also show the output of each function call, if any. **[2]**

(b) Assume further that the partially ordered tree is stored in memory as a heap. Show the heap representation of the *final tree* only, after all of the above function calls. **[1]**

3. Recall the function `MEMBER(x,A)`, which checks for the membership of element `x` in a binary search tree (BST) `A`. In class, we wrote down a recursive version of this function. Now write the pseudocode for a *non-recursive* implementation of it. (You may assume the elements to be integers, and use the same pointer-based BST implementation which was introduced in class; please make sure all your notation / variable names are completely clear.) **[3]**

4. Consider the following cost matrix for a directed graph (infinite cost denotes no direct edge):

	1	**2**	**3**
1	∞	4	1
2	∞	1	4
3	3	9	∞

(a) Draw the corresponding directed graph, also showing the edge weights. **[0.5]**

(b) Run the Floyd-Warshall algorithm on this graph. Show the full A and P matrices at each iteration, starting with A_0 and P as a matrix of all zeroes. **[3.5]**

(c) Recall the recursive function $path(i, j)$, which uses the final P matrix to print out the shortest path between any pair of nodes (i, j). Show the full recursion tree and output resulting from the call $path(2, 1)$. **[1]**

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Section 1. Short Answer Questions

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1. Consider the Towers of Hanoi problem discussed in class:

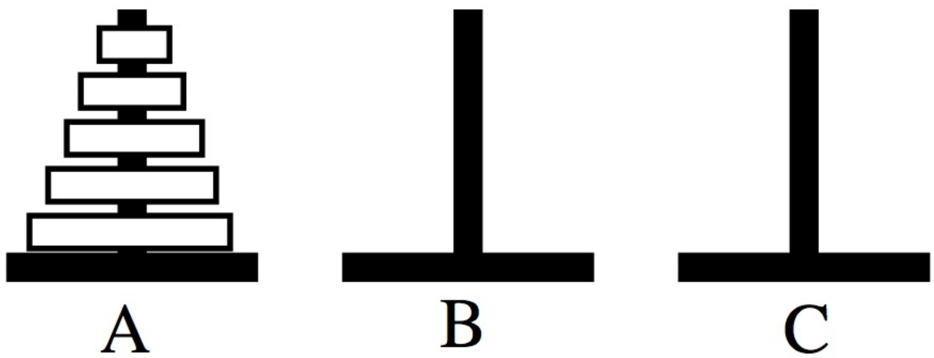

Figure 3: Towers of Hanoi (Copyright J. A. Storer, Brandeis University)

Recall, all the disks are to be moved from Peg A to Peg B, such that a larger disk is never placed on top of a smaller one. Let us define $T(n)$ as the total number of disk moves required to accomplish this, where n is the number of disks. Obtain a simple closed-form expression for $T(n)$. (You may use any approach you wish, but make sure to write out your working and reasoning fully and clearly.) **[4]**

2. Suppose I initialise a new `PRIORITYQUEUE Q`, which is meant to store positive integers such that bigger integers have higher priority. I call the `INSERT` and `DELETEMAX` operations on it in the following sequence:

   - `INSERT(3,Q)`
   - `INSERT(12,Q)`
   - `INSERT(8,Q)`
   - `INSERT(9,Q)`
   - `DELETEMAX(Q)`
   - `INSERT(10,Q)`
   - `INSERT(7,Q)`
   - `DELETEMAX(Q)`

   (a) Assume that the `PRIORITYQUEUE` is implemented using a partially ordered tree. Show what the tree will be after each of the above function calls. Also show the output of each function call, if any. **[2]**

   (b) Assume further that the partially ordered tree is stored in memory as a heap. Show the heap representation of the *final tree* only, after all of the above function calls. **[1]**

3. Recall the function `MEMBER(x,A)`, which checks for the membership of element `x` in a binary search tree (BST) `A`. In class, we wrote down a recursive version of this function. Now write the pseudocode for a *non-recursive* implementation of it. (You may assume the elements to be integers, and use the same pointer-based BST implementation which was introduced in class; please make sure all your notation / variable names are completely clear.) **[3]**

4. Consider the following cost matrix for a directed graph (infinite cost denotes no direct edge):

	1	**2**	**3**
1	∞	5	5
2	3	∞	∞
3	9	5	3

(a) Draw the corresponding directed graph, also showing the edge weights. **[0.5]**

(b) Run the Floyd-Warshall algorithm on this graph. Show the full A and P matrices at each iteration, starting with A_0 and P as a matrix of all zeroes. **[3.5]**

(c) Recall the recursive function $path(i, j)$, which uses the final P matrix to print out the shortest path between any pair of nodes (i, j). Show the full recursion tree and output resulting from the call $path(2, 3)$. **[1]**

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Minor Test II, Form: $\boxed{\text{D}}$ (**please write this Form ID on the cover page of your answer script**)

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Section 1. Short Answer Questions

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1. Consider the Towers of Hanoi problem discussed in class:

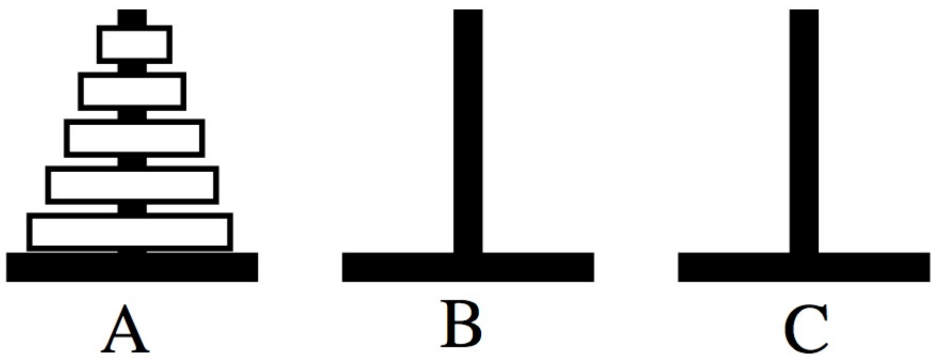

Figure 4: Towers of Hanoi (Copyright J. A. Storer, Brandeis University)

Recall, all the disks are to be moved from Peg A to Peg B, such that a larger disk is never placed on top of a smaller one. Let us define $T(n)$ as the total number of disk moves required to accomplish this, where n is the number of disks. Obtain a simple closed-form expression for $T(n)$. (You may use any approach you wish, but make sure to write out your working and reasoning fully and clearly.) **[4]**

2. Suppose I initialise a new `PRIORITYQUEUE Q`, which is meant to store positive integers such that bigger integers have higher priority. I call the `INSERT` and `DELETEMAX` operations on it in the following sequence:

   - `INSERT(9,Q)`
   - `INSERT(4,Q)`
   - `INSERT(10,Q)`
   - `INSERT(7,Q)`
   - `DELETEMAX(Q)`
   - `INSERT(5,Q)`
   - `INSERT(12,Q)`
   - `DELETEMAX(Q)`

   (a) Assume that the `PRIORITYQUEUE` is implemented using a partially ordered tree. Show what the tree will be after each of the above function calls. Also show the output of each function call, if any. **[2]**

   (b) Assume further that the partially ordered tree is stored in memory as a heap. Show the heap representation of the *final tree* only, after all of the above function calls. **[1]**

3. Recall the function `MEMBER(x,A)`, which checks for the membership of element `x` in a binary search tree (BST) `A`. In class, we wrote down a recursive version of this function. Now write the pseudocode for a *non-recursive* implementation of it. (You may assume the elements to be integers, and use the same pointer-based BST implementation which was introduced in class; please make sure all your notation / variable names are completely clear.) **[3]**

4. Consider the following cost matrix for a directed graph (infinite cost denotes no direct edge):

	1	**2**	**3**
1	4	3	10
2	∞	∞	6
3	6	4	∞

(a) Draw the corresponding directed graph, also showing the edge weights. **[0.5]**

(b) Run the Floyd-Warshall algorithm on this graph. Show the full A and P matrices at each iteration, starting with A_0 and P as a matrix of all zeroes. **[3.5]**

(c) Recall the recursive function $path(i, j)$, which uses the final P matrix to print out the shortest path between any pair of nodes (i, j). Show the full recursion tree and output resulting from the call $path(2, 1)$. **[1]**