[Minor I]

1. we have $N_B = 4$, $p = 0.6$

Natural prior on N is Uniform $(\{1, 2, \dots 10\})$ → ~~uniform~~ constant, cancels out

For posterior:

$$P(N \mid N_B, p) = \frac{\overbrace{P(N_B, p \mid N)}^{P(N_B \mid p, N)\, \cancel{P(p)}}\, \cancel{P(N)}}{\sum_{i=1}^{10} \underbrace{P(N_B, p \mid N = i)}\, \cancel{P(N=i)}}$$

$$= \frac{{}^{N}C_{N_B}\, p^{N_B} (1-p)^{N-N_B}}{\sum_i {}^{i}C_{N_B}\, p^{i} (1-p)^{i-N_B}}$$

we can evaluate the numerator for values of $N = 4, 5, \dots, 10$ (clearly 0 for $N < 4$)

N	$P(N \mid N_B, p)$
4	0.6^4
5	$5 \times 0.6^4 \times 0.4$
6	$15 \times 0.6^4 \times 0.4^2$
7	$35 \times 0.6^4 \times 0.4^3$
8	$70 \times 0.6^4 \times 0.4^4$
9	$126 \times 0.6^4 \times 0.4^5$
10	$210 \times 0.6^4 \times 0.4^6$

(sum)

$= \cancel{0.6^4}(1 + 5 \times 0.4 + 15 \times 0.4^2 + 35 \times 0.4^3 + 70 \times 0.4^4 + 126 \times 0.4^5 + 210 \times 0.4^6)$

$\therefore$ Posterior is:

N	$P(N \mid N_B, p)$	
4	1	
5	2	
6	2.4	
7	2.24	11.5824
8	1.792	
9	1.29024	
10	0.86016	

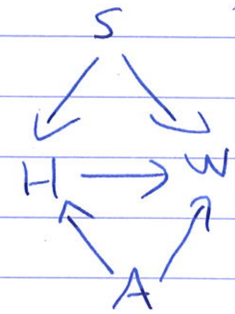
2. $P(\alpha, \beta, \sigma \mid H_i)$

$$= \frac{P(H_i \mid \alpha, \beta, \sigma) \cdot P(\alpha, \beta, \sigma)}{P(H_i)}$$

By definition, H_i has no dependence on $\alpha / \beta / \sigma$: this is what discriminative modelling means / assumes. So :

$$= \frac{P(\cancel{H_i}) \cdot P(\alpha, \beta, \sigma)}{P(\cancel{H_i})} = P(\alpha, \beta, \sigma).$$

3. (a)



(b) $H_i = f(A_i, S_i; \underline{\theta})$, or more precisely, the likelihood from the model will be given by

$P(H_i | A_i, S_i; \underline{\theta})$

Posterior over $\underline{\theta}$ would be

$$P(\underline{\theta} | \underline{A}, \underline{S}, \underline{H}) = \frac{P(\underline{H} | \underline{A}, \underline{S}; \underline{\theta}) P(\underline{\theta})}{P(\underline{H} | \underline{A}, \underline{S})}$$

(using result from Q2)

With the usual IID assumption:

$$P(\underline{\theta} | \underline{A}, \underline{S}, \underline{H}) = \frac{\left[\prod_i P(H_i | A_i, S_i; \underline{\theta})\right] P(\underline{\theta})}{\prod_i P(H_i | A_i, S_i)}$$

$$= \frac{\left[\prod_i P(H_i | A_i, S_i; \underline{\theta})\right] P(\underline{\theta})}{\prod_i \int_{\underline{\theta}} P(H_i | A_i, S_i; \underline{\theta}) P(\underline{\theta}) d\theta}$$