

# Graphical Models

$$P(x) = \frac{1}{Z} \prod_{c \in \mathcal{C}} \phi_{c_j}(x_{c_j})$$

↓  
subset of  $x$   
over nodes in  
clique  $j$

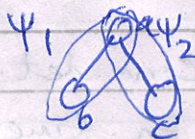
- What are  $\phi_s$ ? Relation to joint distributions?
- How do we choose which (non-minimal) cliques to include?
- Markov Blanket?
- Hammersley - Clifford

$X \perp Z \mid MB(x) \iff$  Joint distr. can be factored over maximal cliques

CI's imposed by network str.



$$P(a, b, c) = P(a)P(b|a)P(c|a, b)$$

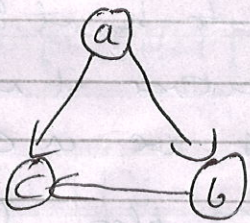


$$P(a, b, c) = P(a)P(b|a)P(c|a)$$

OR

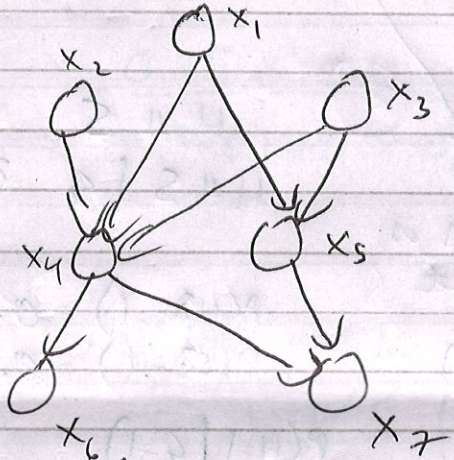
$$= (e.g.) P(a, b)P(c|a) / P(a, c)P(b|c)$$

# Bayesian Nets



(Invitation)

$$p(a, b, c) = p(c) \cdot p(b|a) \cdot p(a|b, c)$$



Directed Acyclic Graph (DAG)

$$p(x_1) p(x_2) p(x_3)$$

$$p(x_4 | x_1, x_2, x_3)$$

$$p(x_5 | x_1, x_3)$$

$$p(x_6 | x_4) p(x_7 | x_4, x_5)$$

$$p(x) = \prod_{k=1}^K p(x_k | pa_k)$$

|  
parents  
of k

*(faint blue text at the bottom of the page)*

# D - Separation

$A \perp\!\!\!\perp B \mid C$  ?

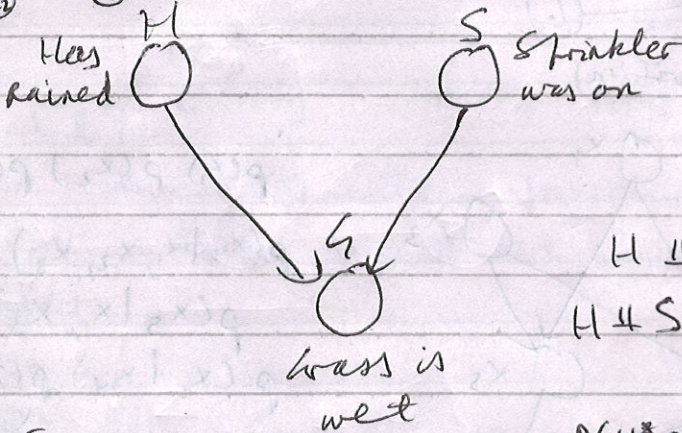
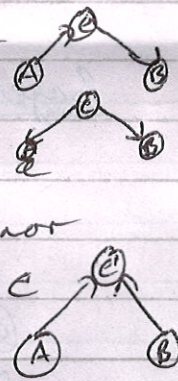
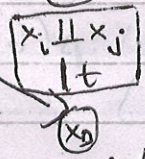
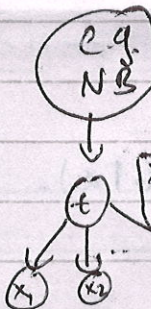
Yes: If all paths from A to B are blocked by C  
blocked by C  $\xrightarrow{\text{tail}(t) \text{ head}(h)}$   
 ↓ Defn.

① Arrows meet h-to-t or

t-to-t at a node in C

② Arrows meet h-to-h at

node, s.t. neither it nor its descendants are in C



$H \perp\!\!\!\perp S$   
 $H \perp\!\!\!\perp S \mid G$  ? x

E.g.

H	S	$P(G=1 \mid H,S)$
0	0	0.1
0	1	0.8
1	0	0.9
1	1	0.95

$P(H=1) = 0.1$   
 $P(S=1) = 0.2$   
 $P(H=1 \mid G=1)$   
 $= \frac{P(G=1 \mid H=1) \cdot P(H=1)}{P(G=1)}$

$$P(G=1) = \sum_H \sum_S P(G=1 | H, S) P(H) P(S)$$

$$= 0.1 \times 0.9 \times 0.8 + 0.8 \times 0.9 \times 0.2 \\ + 0.9 \times 0.1 \times 0.8 + 0.95 \times 0.1 \times 0.2 \\ = 0.307$$

$$P(G=1 | H=1) = \sum_S P(G=1 | H=1, S) P(S)$$

$$= 0.9 \times 0.8 + 0.95 \times 0.2 \\ = 0.91$$

$$\Rightarrow P(H=1 | G=1) = \frac{0.91 \times 0.1}{0.307}$$

$$= \underline{0.296} \quad \text{1st}$$

$$P(H=1 | G=1, S=1)$$

$$= \frac{P(G=1 | S=1, H=1) \cdot P(H=1)}{P(G=1 | S=1)}$$

$$= \frac{0.95 \times 0.1}{0.95 \times 0.1 + 0.8 \times 0.9} = \frac{0.095}{0.815}$$

$$= \underline{0.117}$$

→ Why is this lower?

$$\Rightarrow H \not\perp S | G$$

# Inference

(writing down likelihood of ~~unknown~~ knowns in terms of knowns)

specified forms



$$p(x, y) = p(x) p(y|x)$$

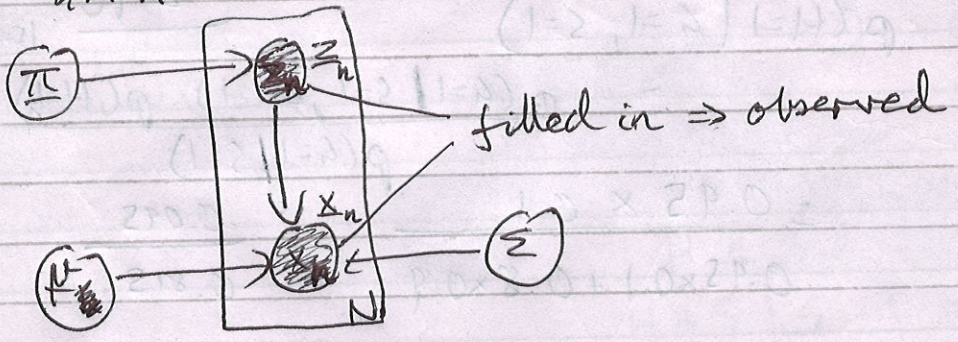
y observed:

$$p(x|y) = \frac{p(y|x) \cdot p(x)}{p(y)}$$

↓  
 $l(x)$   
 (posterior)

$$= \frac{p(y|x) \cdot p(x)}{\sum_{x'} p(y|x') p(x')}$$

EMM:



$$p(x, z | \pi, \mu, \Sigma) \rightarrow l(x, \mu, \Sigma)$$