Hierarchies in representations Latent structure discovery and Dimensionality reduction



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Tacit Knowing (Thinking fast)

Latent Relations

 \rightarrow



DANIEL KAHNEMAN

WINNER OF THE NOBEL PRIZE IN ECONOMICS

Latent Representation



images: 100 x 100 pixels

Manifolds in language

• grounded syntax:



[Nayak and Mukerjee AAAI-12]

Visuo-Motor expertise

in darkened room, works hard to position arm^{*} in a narrow beam of light

> Newborns (10-24 days)

Small weights tied to wrists

Will resist weights to move the arm they can see

Will let it droop if they can't see it



[A. van der Meer, 1997: Keeping the arm in the limelight]

Simulation









Robot self-discovery



discover the world?

Camera Motion and Shape via Factorization

Homogeneous Coordinates

2-D point (x,y) represented as (x₁,x₂,x₃) in homogeneous system

$$x = x_1/x_3$$
 $y = x_2/x_3$

- (x_1, x_2, x_3) same as (kx_1, kx_2, kx_3) same as $(x_1/x_3, x_2/x_3, 1)$
- Similarly for 3-D point (x_1, x_2, x_3, x_4)

Camera Models

- Orthographic
- Weak perspective/Scaled orthographic
- Para-perspective
- Perspective
- Affine
- Projective

Camera Models

- $x = (u,v,1)^T$ or $(x_1,x_2,x_3)^T =>$ homogeneous image coordinates
- $X = (x,y,z,1)^T$ or $(X_1,X_2,X_3,X_4) =>$ homogeneous 3D coordinates

$$\mathbf{x}_{(3x1)} = \mathbf{T}_{(3x4)} \mathbf{X}_{(4x1)}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Orthographic Camera Model



u=x v=y

Orthographic Camera Model

$$\mathbf{T}_{orth} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Perspective Camera Model



Perspective Camera Model

$$\mathbf{T}_p = \left[\begin{array}{rrrr} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{array} \right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Weak Perspective or Scaled Orthographic

$$\mathbf{T}_{wp} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Z_{ave}/f \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & Z_{ave}/f \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Affine Camera Model

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ 0 & 0 & 0 & T_{34} \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ 0 & 0 & 0 & T_{34} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

Projective Camera

$$\mathbf{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} \\ T_{21} & T_{22} & T_{23} & T_{24} \\ T_{31} & T_{32} & T_{33} & T_{34} \end{bmatrix}$$

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Structure from Motion by Factorization

Tomasi, Carlo, and Takeo Kanade. 1992 "Shape and motion from image streams under orthography: a factorization method.

Factorization Method

- Given: Image stream, where P points have been tracked over F frames
- Image coordinates of p-th point in f-th frame = u_{fp}, v_{fp}
- **W** = 2F x P Measurement Matrix
- Column => observation for point Row => observation for frame

$$W = \begin{bmatrix} u_{11} & \dots & u_{1P} \\ \dots & \dots & \dots \\ u_{F1} & \dots & u_{FP} \\ v_{11} & \dots & v_{1P} \\ \dots & \dots & \dots \\ v_{F1} & \dots & v_{FP} \end{bmatrix}$$

Factorization Method : Orthography

- $\mathbf{W} = \mathbf{M}_{(2Fx3)}\mathbf{S}_{(3xP)} + \mathbf{t}_{(2Fx1)}[1...1]_{(1xP)}$ translation vector **t**: element f = mean of row f in **W**
- registered measurement matrix W*
 W* = W t [1...1] = M S (rank 3)
- **M**_(2Fx3) : row f = camera horiz/ vert axes in frame f**S**_(3xP) : column p = (x,y,z) of tracked point p

OBJECTIVE : obtain **M** and **S** from W* via SVD

Hotel Stream Data



Hotel Stream Data



frame 50

Hotel Stream Data



frame 100

Tracked Features



Tracking



Factorizataion Method : Orthography

- Creation of measurement matrix **W**
- Obtaining registered measurement matrix **W***
- Performing SVD to obtain
 W* = UEV^T = MS
- $M = UE^{0.5}$ $S = E^{0.5}V^T$...not unique solution
- MS or $(M_2A)(A^{-1}S_2)$...both are solutions
- Constraints to obtain unique A
- Directions can be aligned to camera directions in first frame

SVD



Camera Motion Estimation [M]



Camera Motion Estimation [M] camera pitch (degrees)



3D Shape Estimation [S]



Multi-body Factorization Method

- Multiple objects in a video sequence moving independently
- Earlier approach => from object frame (as if object stationary and camera moving)
- Common representation required for all objects in multiobject scenario
- Why not look from **camera frame**!!!
- (Moving camera+static object) equivalent to (static camera +moving object)
- unique for all objects for a frame

Multi-body Factorization Method

- Points from all objects collectively taken
- Expected Rank <= 4*number of objects
- Shape Interaction Matrix

$\mathbf{Q} = \mathbf{V}\mathbf{V}^{\mathsf{T}}$

- Q independent of camera orientation, coordinate transformation
- Permuting columns of V does not change values of Q matrix(only row/column number changed in same way as V permutation)
- Only the interaction values of points from same object get non-zero value.
- Block-diagonalization to obtain the points corresponding to same object
Multi-body Factorization Method



Aspects of Factorization

- All images treated uniformly
- Alternate approaches are initialization based and may do poorly if improper.
- Convergence guaranteed by numerical approach
- In multi-object factorization, can determine number of objects and point clusters

Unsupervised Action Learning and Anomaly Detection

3.2. Topic Modelling

• Given: Document and Vocabulary

* Document is histogram over vocabulary

• Goal: Identify topics in a given set of Documents

Idea: Topics are latent variables

Alternate view :

- Clustering in topic space
- Dimensionality reduction

3.3. Models in practice

• LSA

Non-parametric clustering into topics using SVD.

• *pLSA* :

Learns probability distribution over fixed number of topics; Graphical model based approach.

• LDA :

Extension of pLSA with dirichlet prior for topic distribution. Fully generative model.

4. Vision to NLP : Notations

Text Analysis	Video Analysis
Vocabulary of words	Vocabulary of visual words
Text documents	Video clips
Topics	Actions/Events

5.1. Video Clips

- 45 minute video footage of traffic available
- 25 frames per second
- 4 kinds of anomaly
- Divided into clips of fixed size of say 4 6 seconds

5.2. Visual Words

- Each frame is 288 x 360
- Frame is divided into 15 x 18 parts, each part containing 400 pixels
- Features
 - Optical flow
 - Object size
- Background subtraction is performed on each frame to obtain the objects in foreground. Features are computed only for these objects
- Foreground objects consist of vehicles, pedestrians and cyclists

5.2. Visual Words (contd..)

- Foreground pixels then divided into "big" and "small" blobs (connected components)
- Optical flow computed on foreground
- Flow vector quantised into 5 values :
 - Static
 - Dynamic- up, down, left and right
- 15x28x5x2 different "words" obtained

5.3. Foreground Extraction





5.4. Optical Flow: Heat Map









6. Modelling pLSA

- Training Dataset: no or very less "anomaly"
 - Test Dataset: usual + anomalous events

Procedure

- Learn P(w|z) and P(z|d) from training data
- Keeping P(w|z), estimate P(z|d) on test data
- Threshold on likelihood estimate of individual test video clips

7. Results Demo

• 3 clips

• 3 different types of anomalies

8. *Results (ROC plot)*



8. Results (PR curve)



Motion Planning

Motion planning

Given start / goal image, map to manifold using local interpolation

Use k-nn connectivity in manifold as "roadmap" for motion planning



Obstacle modeling by node deletion



If obstacle intersects robot in image space → delete corresponding nodes from "visual roadmap"

Path planning as obstacle moves



Mobile robots



















Residual error : disk robot



Robot Motion Planning



Path Planning Interface



Path 2: 2316 -> 41 -> 328 -> 50 -> 2257 -> 224 -> 1436 -> 657 -> 263 -> 1854 -> 1608 -> 2690 -> 1524 -> 1315 -> 843 -> 323 -> 2226 -> 2061 -> 911 -> 399 -> 796 -> 1817 -> 1352 -> 465 -> 386 -> 786 -> 1994 -> 1761 -> 1983 -> 2047 -> 2650 -> 450 -> 1400 -> 2700 -> 2535 -> 1038 -> 1634 -> 45 -> 1093 -> 1609 -> 2369 -> 828 -> 597 -> 184





Real robots

SCARA arm





SCARA arm : degrees of freedom



Background Subtraction : Robot





image

foreground (moving part)



learned background

Background Subtraction : Obstacle





Visual Configuration Space





Application to Graphics

Head Motion



[M. S. Ramaiah, Ankit Vijay, Geetika Sharma, Amitabha Mukerjee IEEE VR-13]

Minimal Commitment Language Acquisition
Previous Work: Unsupervised Semantics

- single word or phrase learning [no grammar]
 - Hand-coded propositional (T/F) semantics
 - [plunkett etal 92]
 - [regier 96] (prepositions)
 - [steels 03] [roy/reiter 05] [caza/knott 12]
 - Supervised Learning of semantics
 - [kate/mooney 06] : set of predicates are known
 - [yu/ballard 07] : semantics = scene-region
- Unsupervised Semantic Acquisition : "right" granularity for concepts; dynamic predicates

What is a Symbol?

• grounded **lexicon**:





• grounded lexicon:



semantic pole : perceptual patterns (image schemas)
 → probabilistic predicate + arguments

Grammar

• grounded syntax:



Minimal Commitment

- minimize prior knowledge in agent:
 - preference: minimize description lengths
 → inventory of machine learning algorithms
 - no knowledge of grammar no POS tags, no syntactic structure
 - no knowledge of domain
- **bootstrapping** stage:
 - semantic schemas come first
 - language regularities later

POS categories are discovered



Acquisition Domains

Previous Work: Unsupervised Semantics

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- Unsupervised Semantic Acquisition : "right" granularity for concepts; dynamic predicates

Previous Work: Grammar

- Grammar learning:
 - Grammatical categories:
 - [redington etal 98] (RNN)
 - [wang / mintz 07] (frequent frame)
 - Grammar induction : Structure is known
 - No semantics:
 - [marino etal 07] [solan edelman 05]
 - Propositional semantics
 - [kwiatkowski zettlemoyer 10] (SVM)
 - [kim/moonev 12] (altered visual input)

Language Acquisition : Domain 1

• Perceptual input



- Discovery Targets:
 - semantics: objects, 2-agent actions, relations
 - lexicon : nominal, transitive verbs, preposition
 - lexical categories: N VT P Adj
 - constructions: PP VP S
 - sense extension (metaphor) [nayak/mukerjee (AAAI-12)]

Language Acquisition : Domain 2

• Perceptual input



- Discovery Targets:
 - semantics: object categories, motion categories

Language Acquisition : Domain 2

 object categories



- Discovery Targets:
 - semantics: object categories, motion categories
 - lexicon : word boundaries, nominals, intransitive verbs
 - construction · intransitive VP

Video Fragment



Discovering Language

- Perceptual structure discovery:
 - Given perceptual space W discover set of structures
 Γ that partition it into patterns relevant to agents goals.
 - Elements $\gamma \in \Gamma$ constitute a hierarchy; structures learned earlier are used for more complex patterns
- Linguistic Structure Discovery
 - Given set of sentences formed from words w ∈L, discover set of subsequences A that result in a more compact description of the structure
 - Elements λ ∈ Λ constitute a hierarchy, leaf nodes (POS) are subsets of L

Semantics First: Objects / Nominals

Language Grounding: Entity/Object

object = coherent salient region in perceptual space

- o object view schema [white maruti 800 from camera 1]
- o object schema [white maruti 800]
- object category schema [car]
- o bottom-up dynamic attention



Language – Meaning Associaction

• Relative Association (bayesian)

$$P(\gamma_j|\lambda_i) = \frac{P(\lambda_i|\gamma_j)P(\gamma_j)}{P(\lambda_i)} \propto \frac{P(\lambda_i|\gamma_j)}{P(\lambda_i)}$$

• Mutual association (contribution to M.I.)

$$P(\lambda_i, \gamma_j) \log \frac{P(\lambda_i, \gamma_j)}{P(\lambda_i) P(\gamma_j)}$$
$$I(\Gamma, \Lambda) = \sum_i \sum_j P(\lambda_i, \gamma_j) \log \frac{P(\lambda_i, \gamma_j)}{P(\lambda_i) P(\gamma_j)}$$

Language Grounding: Nominals

[BS]			[\$\$]			[C]		
word(s)	A_{ij}^{rel}	A_{ij}^{mut}	word(s)	A_{ij}^{rel}	A_{ij}^{mut}	word(s)	A_{ij}^{rel}	A_{ij}^{mut}
square	0.70	1.41	little	0.66	0.79	circle	0.79	2.11
big	0.89	1.11	small	0.72	0.63	square	0.41	1.54
box	0.69	0.78	square	0.46	1.12	little	0.68	1.22
the big	0.87	0.71	small square	0.93	0.53	the little	0.71	0.81
big square	0.94	0.75	little square	0.89	0.46	little circle	0.91	0.60
large square	0.86	0.15	the little	0.70	0.54	the big	0.48	0.61

Perceptual Discovery : Actions : Verbs

Perceptual Discovery: 2-agent actions

Consider every pair of objects A,B
 A : attended to object (tr)
 B : other object (landmark, lm).

• 2 features suffice:

relative-velocity and relative position $pos \cdot velDiff : (\vec{x}_B - \vec{x}_A) \cdot (\vec{v}_B - \vec{v}_A)$ relative pose and the sum of the velocities $pos \cdot velSum : (\vec{x}_B - \vec{x}_A) \cdot (\vec{v}_B + \vec{v}_A)$



Perceptual Discovery: 2-agent actions

• Static time-shots of feature space trajectories



Switching the in-Focus agent

□ Human Labels (CC, MA, Chase) → Ground Truth
□ Label Vs Cluster assigned



Number of Clusters from MNG = 4 when *Edge Aging* = 30 (0.9 prob)

CC: Come-Closer (C1), MA: Move Away (C2), $C_3 \& C_4$: Chase **Chase sub-categories**: Chase_*RO-chases-LO*: $C_3 \rightarrow$

Chase_LO-chases-RO: $C_4 \rightarrow$



Hierarchy in Concept Space

 \Box More clusters \rightarrow Reveals category hierarchy:



Similarly: Move-Away : 3 subclasses

Number of Clusters from MNG = 8 when *Edge Aging* = 16



 C_1 , C_5 , C_6 : sub-classes of *Come-Closer*, C_2 , C_7 , C_8 :of *Move-Away*

Two agent action ontology



Learning verbs

CLUSTER 1 (Come-Close)			CLUSTER 2 (Move-Away)			CLUST (Cha	TER 3 (se)	CLUS (Ch	CLUSTER 4 (Chase)		
ONE WORD LONG LINGUISTIC LABELS(MONOGRAMS)											
corner	0.077		away	0.069		chase	0.671	chase	0.429		
move	0.055		move	0.055		other	0.185	after	0.112		
attack	0.042	I	chase	0.049		around	0.183	out	0.033		
	TWO WORD LONG LINGUISTIC LABELS(BIGRAMS)										
each other	0.086		move away	0.111		chase around	0.306	chase after	0.218		
move toward	0.065		go into	0.035		each other	0.227	just chase	0.060		
toward each	0.065		into with	0.035		chase each	0.198	chase out	0.058		
THREE WORD LONG LINGUISTIC LABELS(TRIGRAMS)											
move toward each	0.182		go into with	0.099		chase each other	0.558	just chase out	0.142		
toward each other	0.182		run away out	0.051		start run away	0.132	run away out	0.047		
move close together	0.114		scare in corner	0.032		begin to move	0.127	to go after	0.031		

Discovering Containment Relations : Prepositions

Clustering spatial relations

Feature Commitment: Visual angle subtended at trajector by landmark



Histogram of visual subtended angle for the 3 shapes



Clustering spatial relations

IN cluster (emergent)





Words for motions ending in / out





IN - Containment

IN	A ^{rel}	A_{ij}^{mut}	INTO	A ^{rel}	A_{ij}^{mut}	OUT OF	A ^{rel}	A_{ij}^{mut}
inside	0.79	11.78	into	0.82	6.98	out	0.65	5.71
into	0.90	9.43	inside	0.53	1.03	leaves	1.00	4.16
in	0.61	4.16	enters	1.00	4.85	exits	1.00	3.46

Syntax discovery and Semantic Association

Syntax Discovery

- Syntactic discovery:
 - Given input text, attempt to find graph that results in minimizing the description length
 - Relational Graph RDS: patterns as nodes; edges as transitions
 - Attempt to edit RDS to detect significant patterns
 - Equivalence classes emerge at the nodes

$$\begin{bmatrix} in \\ inside \\ inside \\ into \end{bmatrix} \rightarrow the \rightarrow box$$

ADIOS [solan / edelman 05]

Computing the Image Schema

Our reflective baby has discovered: "in" = label corresponding to this image schema

Hence: symbol for [IN] is

(note: this is an early, very basic, low-confidence characterization



Language Structures : Verbs

$$1. \left[\begin{array}{c} the \rightarrow \begin{bmatrix} big \\ large \\ the \rightarrow square \end{array} \right] \rightarrow square \left[\begin{array}{c} scares \\ approaches \\ chases \end{array} \right] \rightarrow \left[the \rightarrow \begin{bmatrix} small \\ little \end{array} \right] \right]$$

$$2. \begin{bmatrix} ball \\ box \\ door \\ square \end{bmatrix} \rightarrow \begin{bmatrix} moved \\ moves \\ runs \end{bmatrix}$$

ADIOS [solan / edelman 05]

Hindi Acquisition: Word learning

[BS]		[SS]			[C]			[IN]			
word(s)	A_{ij}^{rel}	A^m_{ij}	word(s)	A_{ij}^{rel}	A^m_{ij}	word(s)	A_{ij}^{rel}	A^m_{ij}	word(s)	A_{ij}^{rel}	A^m_{ij}
बक्सा	.77	.37	बक्सा	.62 .44		गौला	.83	.54	अन्दर	.80	1.30
baksA/box			baksA/box			golA/ball			andar/in		
बडा(badA/	.85	.18	छोटा(chota/	.90	.25	बक्से	.63	.27	बाहर (bA-	.78	.73
big) बक्सा			small) बक्सा			के(ke/-)			har/out)		

Incipient Syntax
Scaling up?

- Domain-specific grammar
- After learning several such grammars for different domains, how to merge?

Conclusion

The power of Latent Discovery

- Data is not randomly drawn
- Discovering implicit strucgture in data \rightarrow
 - Tacit Knowledge
- Much work remains in scaling up
 - How to merge diverse domains?
- •Learning to plan motions
 - Learn low-dimensional representations of motion
- Learning Language / Vision

Humans and Robots



madhur ambastha cs665 2002