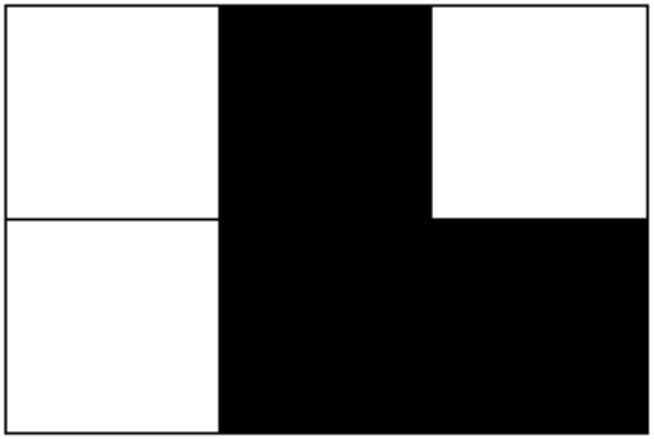
EEL806: Practice Sheet I

September 22, 2013

1. Let us consider a slightly modified version of the toy example for segmentation discussed in class. Suppose we have the image below, which has 6 pixels which each have one of two intensity values: Black (B) or White (W), i.e., $x_i \in \{B, W\}$.



For segmentation, we set up a hidden variable model, assuming that each pixel has an associated *world state* w_i, which we seek to infer. We will assume a binary segmentation, i.e., $w_i \in \{0, 1\}$. Further, we will take the joint distribution of the x_i and w_i values to be modelled by a standard Markov Random Field energy function for a 4-neighbourhood graph:

$$E(W, X) = \sum_{i=1}^{6} \psi_1(w_i; x_i) + \sum_{(i,j) \in Edges} \psi_2(w_i, w_j; x_i, x_j). \quad (1)$$

Here *Edges* denotes the set of edges in the graph.

Suppose further that we have already defined/learnt the potential functions ψ_1 and ψ_2:

$\mathbf{x_i}$	$\boldsymbol{\psi_1}(\mathbf{w_i = 1}; \mathbf{x_i})$	$\boldsymbol{\psi_1}(\mathbf{w_i = 0}; \mathbf{x_i})$
B	0.9	0.1
W	0.1	0.9

So far everything is exactly the same as the example discussed in class. Now, let us alter the definition of ψ_2 a little:

$\mathbf{x_i}$	$\mathbf{x_j}$	$\boldsymbol{\psi_2}(\mathbf{w_i = w_j}; \mathbf{x_i}, \mathbf{x_j})$	$\boldsymbol{\psi_2}(\mathbf{w_i \neq w_j}; \mathbf{x_i}, \mathbf{x_j})$
B	B	0.1	0.9
B	W	0.2	0.8
W	B	0.2	0.8
W	W	0.1	0.9

(a) Given the above definitions of the potential functions ψ_1 and ψ_2, we now wish to infer the most likely world states for the 6 pixels in the given image. This corresponds to minimising the energy function (1) with respect to W. Convert this minimisation problem into a min-cut problem: draw the augmented MRF graph, with the source and sink nodes, and all the edges weighted appropriately.

[Note: There was an error in the way some of the weights were written in class. Specifically, the weights assigned to the inter-pixel edges (as opposed to the edges connecting a pixel to source or sink) were incorrect. Try to figure out this error and correct it here.]

(b) Figure out the solution to the min-cut problem: what are the optimal segment labels for each of the six pixels?

(c) Is this different from the optimal solution obtained when ψ_2 is defined the way it was done in class? Which definition of ψ_2 better enforces the smoothness property for the inferred segmentation?