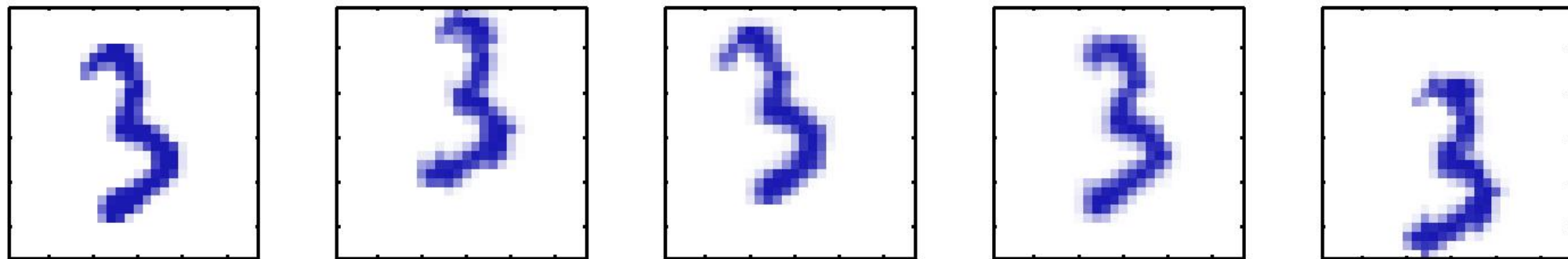


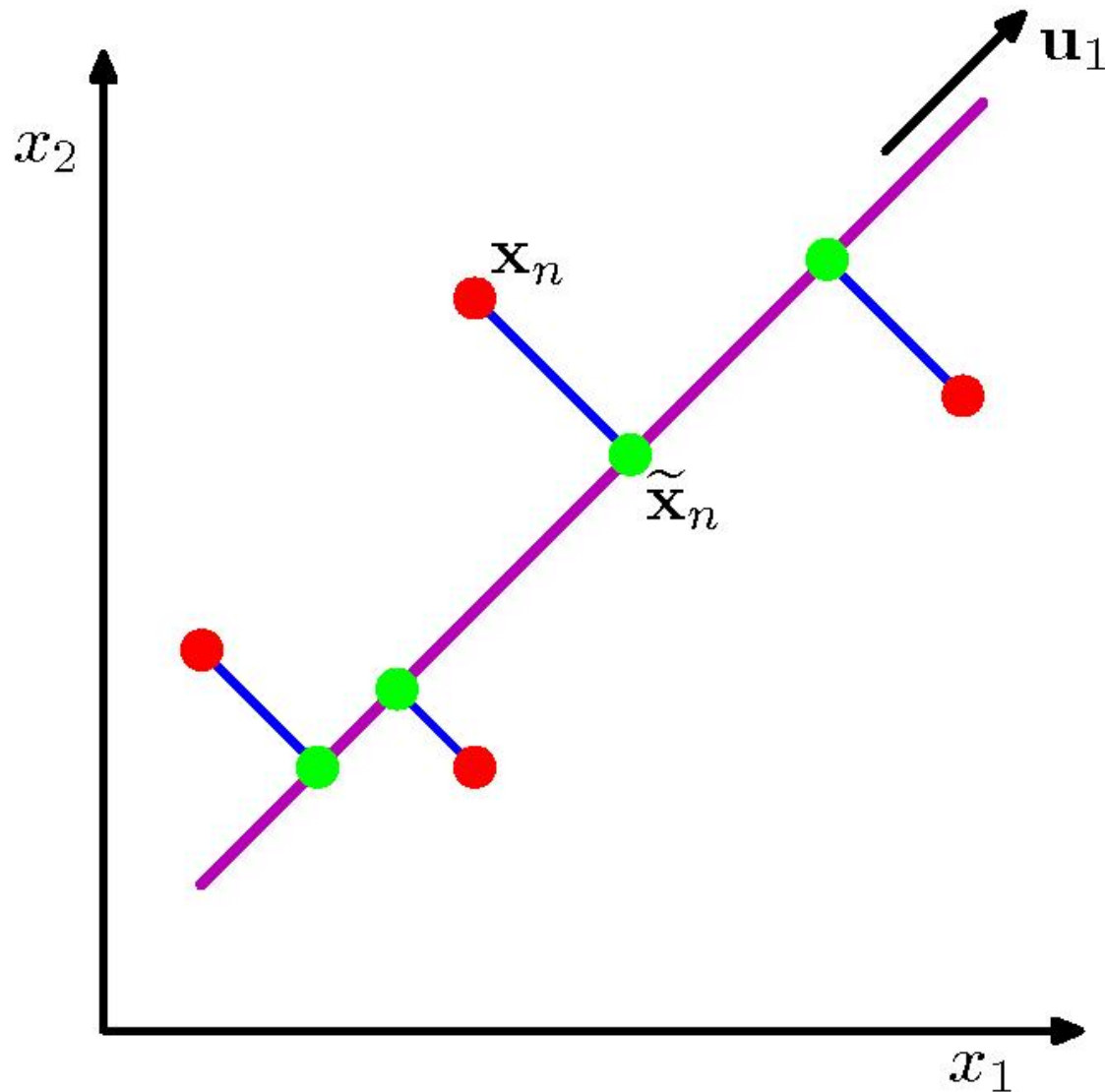
If all 3s were made exactly the same, except for translation and rotation...



Intrinsic dimensionality is only 3, despite  
 $100 \times 100 = 10,000$  pixels

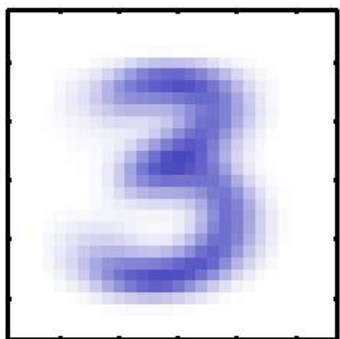
Projection of 2-D points (red) onto 1-D (magenta line) via PCA, such that the variance of the projected points (green) is maximised

Also, equivalent to minimising sum-of-squares of projection errors (blue lines)

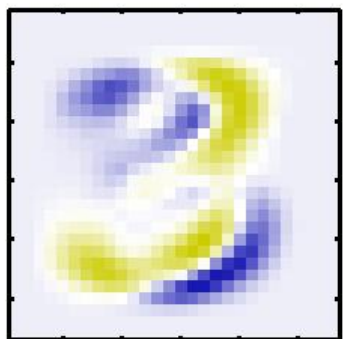


# PCA on data set of images of digit 3 (blue = positive, yellow = negative)

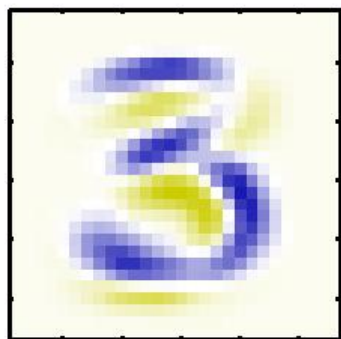
Mean



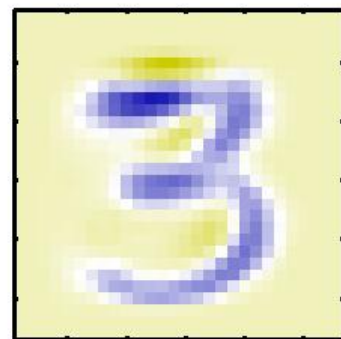
$\lambda_1 = 3.4 \cdot 10^5$



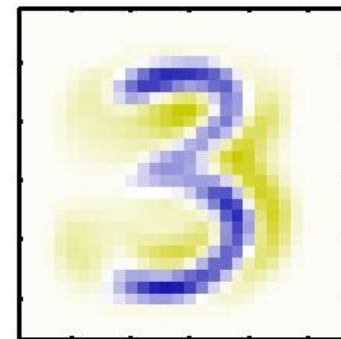
$\lambda_2 = 2.8 \cdot 10^5$



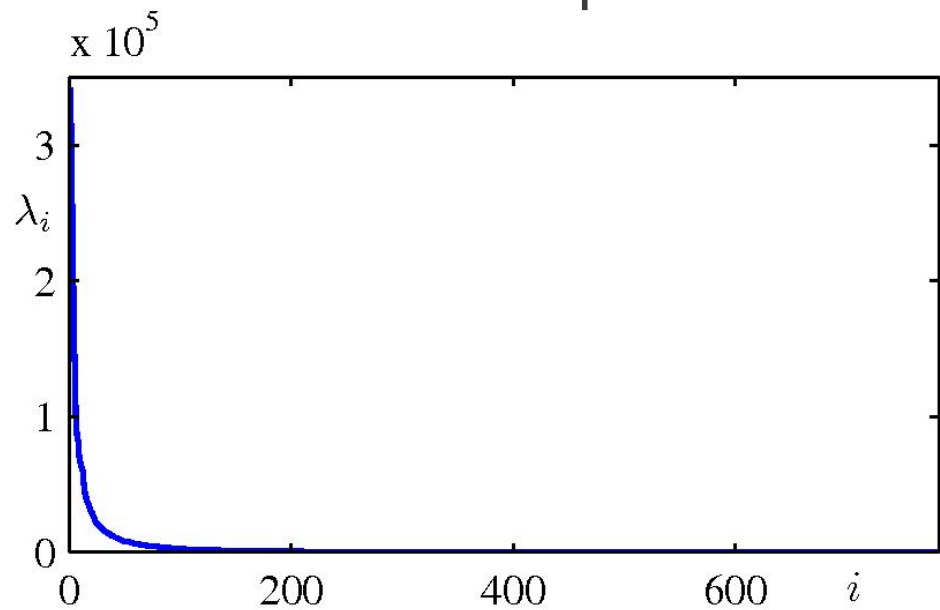
$\lambda_3 = 2.4 \cdot 10^5$



$\lambda_4 = 1.6 \cdot 10^5$

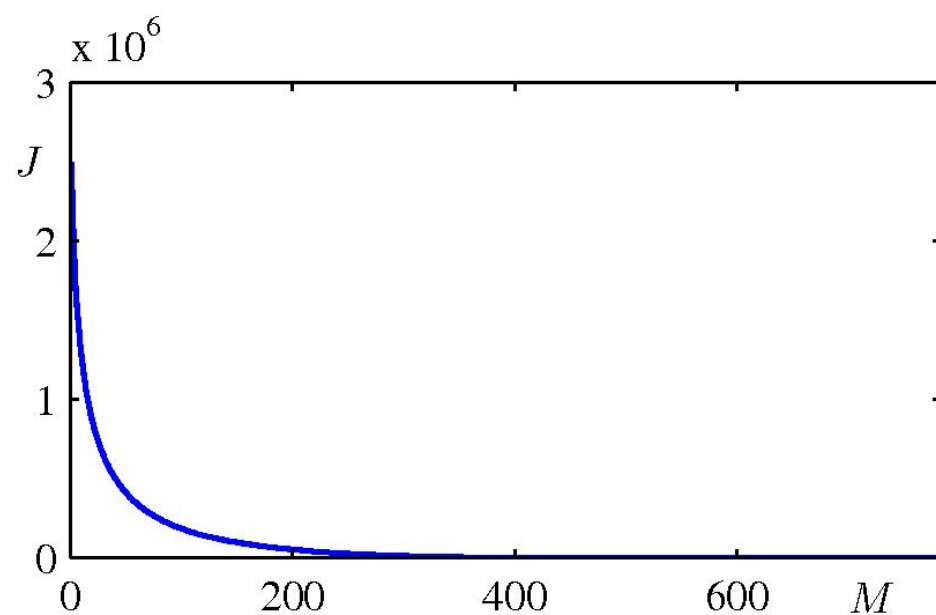


Variance captured



(a)

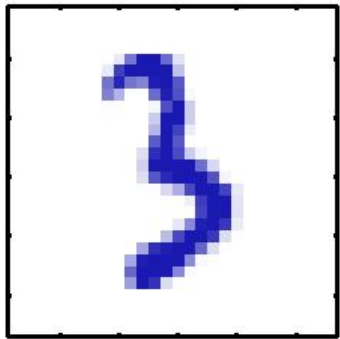
Residual variance



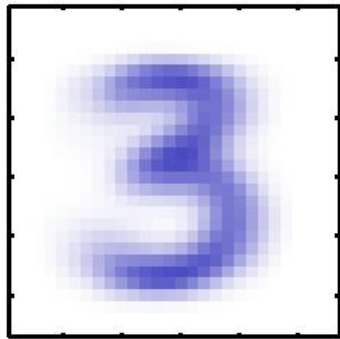
(b)

# Example PCA reconstruction with increasing M

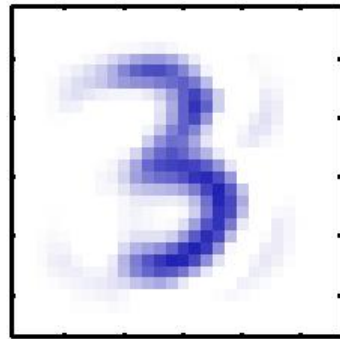
Original



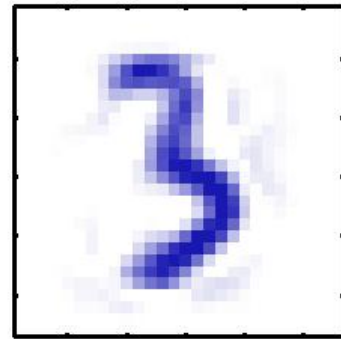
$M = 1$



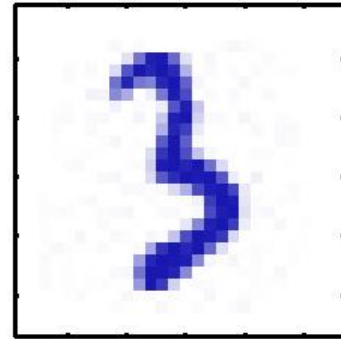
$M = 10$

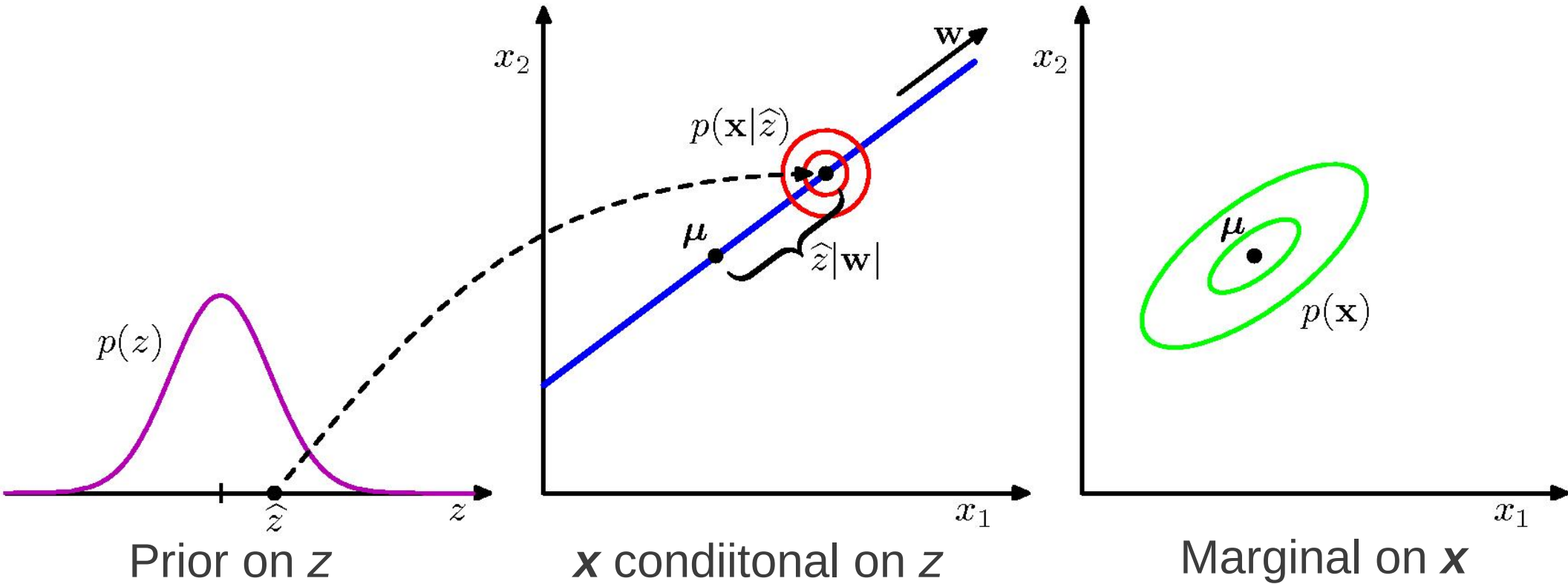


$M = 50$



$M = 250$

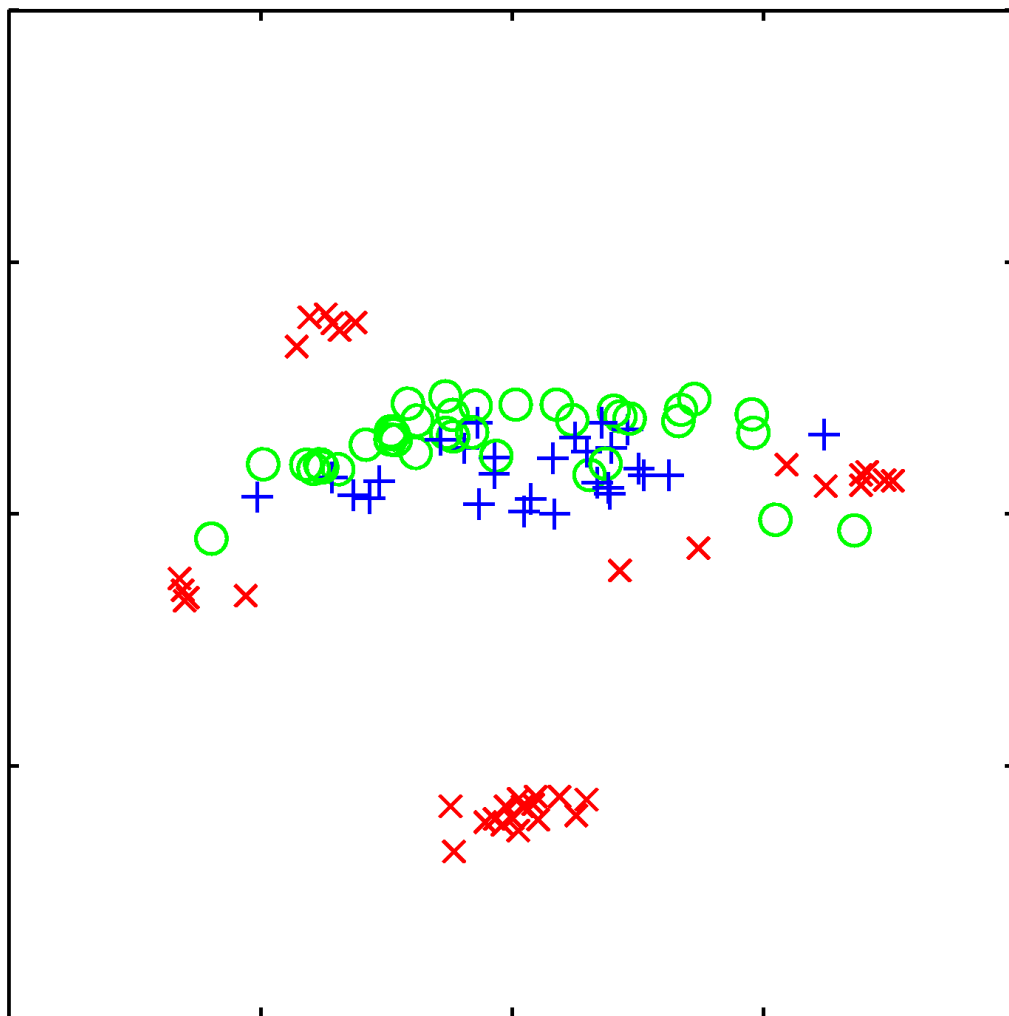




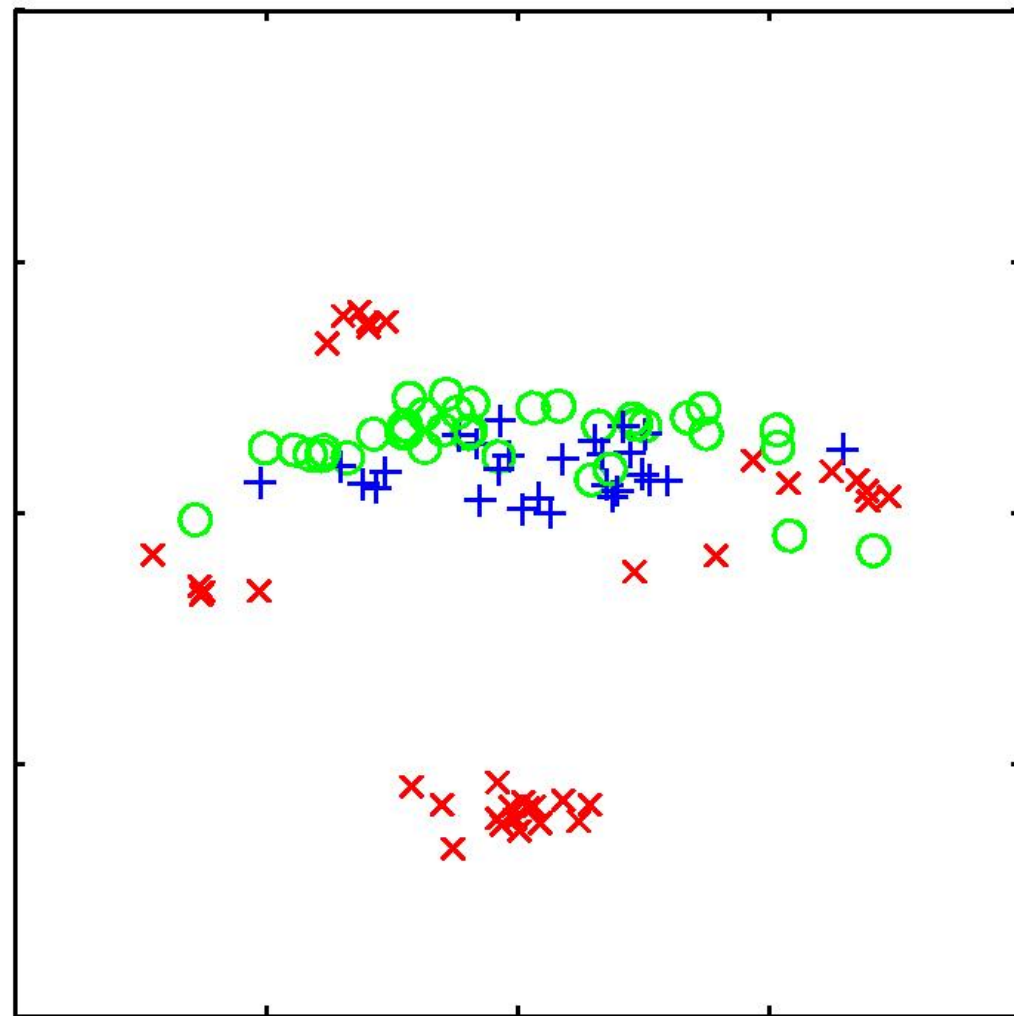
1-D to 2-D probabilistic PCA (i.e., factor analysis with isotropic covariance matrix) as a generative model.

# Probabilistic PCA via EM can also deal with missing values

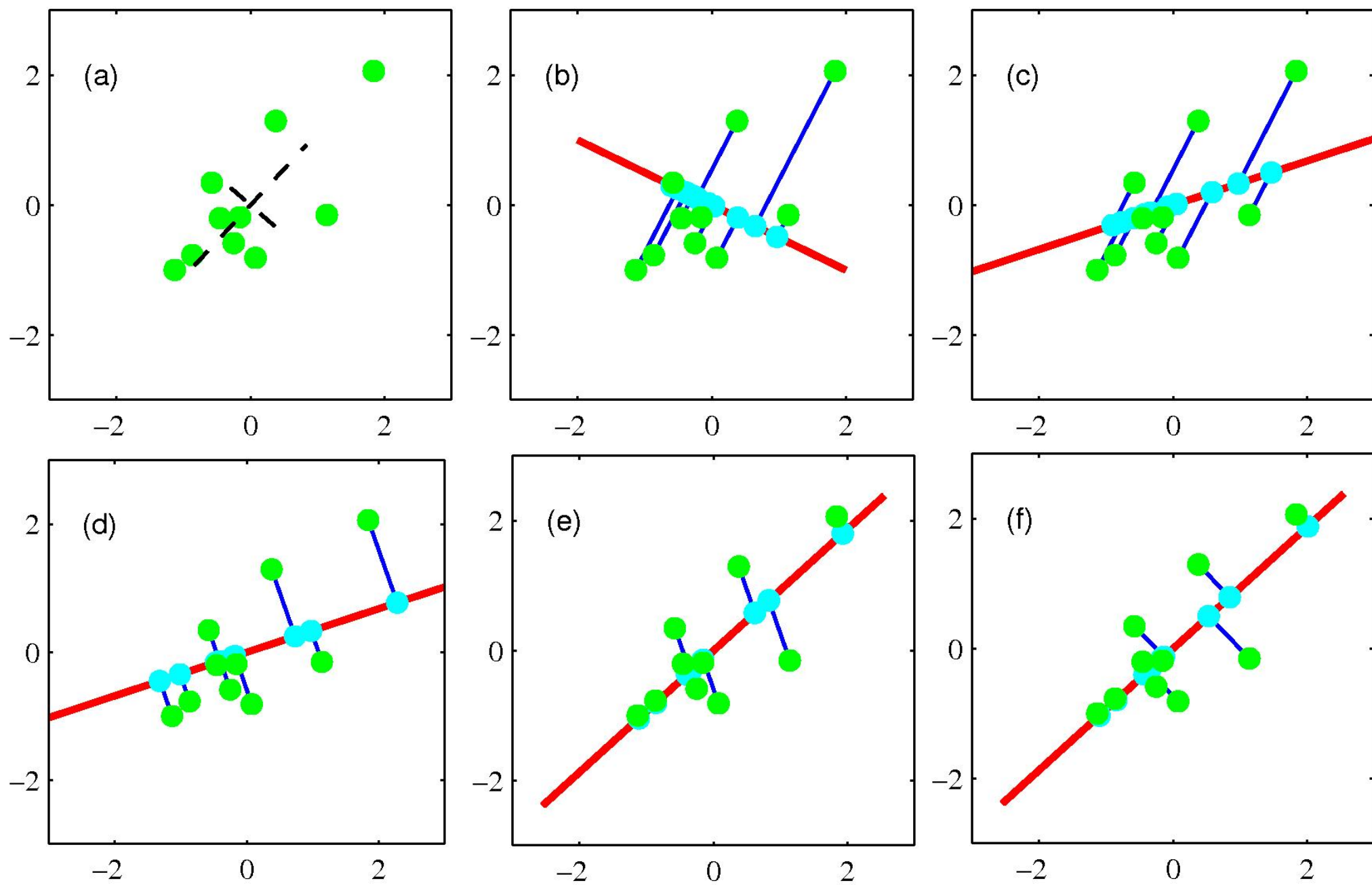
## Full data PCA



## PCA with 30% missing values



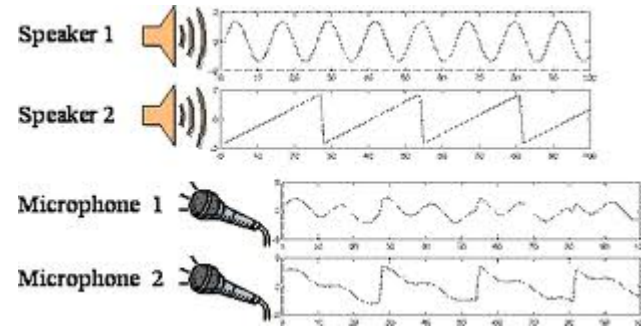
# EM algorithm for probabilistic PCA



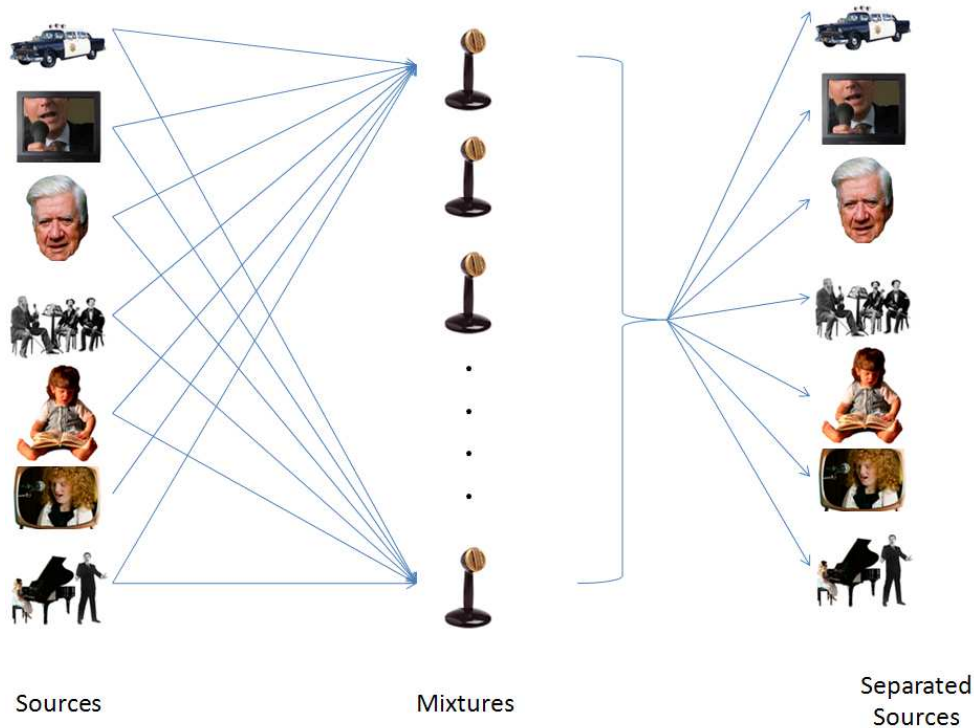
# Cocktail party problem; or, blind source separation



[yukuan.blogspot.com]



[www.comp.lancs.ac.uk]



$$\mathbf{x} = \mathbf{A}\mathbf{z}$$

$\mathbf{z}$ : Source

vector

$\mathbf{x}$ : Observation

vector

$\mathbf{A}$ : Mixing

matrix

[onionesquereality.wordpress.com]