

Throughput Modeling of Multihop Radial Wireless Sensor Networks

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Abstract—This paper presents an analytic model for throughput characterization in a single sink multihop wireless sensor network. The numerical results show that, for a given node density, due to multiaccess contention and nodal buffer limit, the network attains a saturation throughput. As a result, maximum number of hops to the sink is limited by the data traffic generation rate, or conversely, maximum tolerable traffic generation rate is limited by the number of hops to the sink. The eventual goal of this performance modeling has been to use the knowledge of data handling rate at a node for determining nodal energy consumption, and hence predicting the possibility of network partitioning. Additionally, the developed model can be used for sensor network design and deployment optimizations.

I. INTRODUCTION AND MOTIVATION

In a distributed networked sensing task, collected data at the field sensors are forwarded to a predetermined sink node – which could be a remote monitoring and control center, or a clusterhead in a hierarchical sensor network. Multihop nature of forwarding implies that, in addition to forwarding their own data, the intermediate nodes will be responsible for relaying data of the peripheral nodes. Field data from multiple nodes are accumulated (in uncompressed or compressed form) as the sink node is approached, and as a result the traffic flow intensity increases in the same direction. As multiple nodes try to forward data to one node at an intermediate stage, contention to access the wireless channel occurs among them. This leads to queueing/dropping of packets at the intermediate nodes. It is important to quantify the effect of such contentions on the multihop forwarding performance.

Many-to-one forwarding in sensor networks has been studied from congestion control fairness perspectives [1], where transport layer scheduling is suitably controlled to avoid packet dropping as they propagate toward the sink. To enable successful many-to-one forwarding, access scheduling mechanisms and buffer requirements were investigated in [2]. Another set of work proposed medium access control (MAC) level solutions to resource constrained multihop sensor networks (e.g., [3], [4], [5]). While the prior works addressed the important issue of data forwarding constraints, to the best of our knowledge, a quantitative basis for a tolerable network (or cluster) size in many-to-one multihop networks is still missing.

Systems implementation level many-to-one multihop forwarding constraints, such as message length and buffer size, were studied in [6], where the number of message transmissions was reduced by in-network aggregation. It also showed

how the data forwarding performance drastically decreases with increase in hop length, but did not address the effect of multiaccess constraints. A few recent MAC level analytic performance models of CSMA/CA (carrier sense multiple access with collision avoidance) sensor networks include [7], [8], [9], [10]. [9] considered star network topology with beacon-enabled nodes and two-way communication. [10] investigated multiaccess contention in a many-to-one data aggregation scenario with beacon-enabled nodes, where the nodes were modeled such that the field nodes are always in the transmit state and the sink is in the receive state. All of these analytic works addressed the single-hop aggregation problem.

In the current work, we present a stochastic model to characterize the throughput of a multihop many-to-one sensor network with uniformly distributed field nodes. The developed model is used to study the effect of network traffic generation rate on the allowable number of forwarding hops, or conversely, for a given maximum number of hops to the sink, maximum allowable traffic generation rate at the nodes. For example, our numerical results show that, at a per-node new traffic generation rate 0.002 per data packet duration and with 4-to-1 forwarding in each stage (hop), the maximum allowable number of hops up to where the carried traffic continue to grow from the previous stage is only 6.

In Section II the multihop forwarding scenario is outlined. Section III contains the developed stochastic model. Numerical results are presented and some key observations are made in Section IV. The paper is concluded in Section V.

II. MULTIHOP MANY-TO-ONE FORWARDING PROBLEM

We consider a single cell scenario with a single sink, which could be the entire network or a single cluster. A typical case of many-to-one (radial) multihop forwarding is depicted in Fig. 1, where, for analytic tractability, a circular network

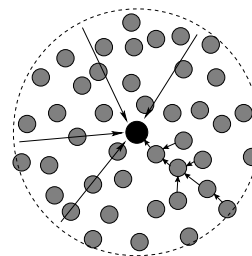


Fig. 1. Field nodes use multihop strategy to forward data to the central sink represented by a dark circle.

space is considered with randomly distributed field nodes and a centrally located sink.

Contrary to the single-hop radial forwarding, multihop forwarding requires that a node in will have to switch between two states: transmit and receive.* IEEE 802.15.4 WPAN (wireless personal area network) standard [11] has been chosen as the base reference for specifying the system. The multihop forwarding capability requires that all field nodes are Full Function Devices, i.e., each node is capable of generating its own data traffic as well as communicating with its neighboring peer nodes as a relay. We will use the results of isolated one stage throughput analysis in [10] to study the impact of radial multihop forwarding on the achievable throughput.

III. THE RADIAL FORWARDING THROUGHPUT MODEL

In this section, we develop an analytic framework for the multihop radial forwarding throughput characterization.

A. System specifications and assumptions

To develop the throughput model for multihop radial forwarding, the useful standard specifications and assumptions drawn on the network are as follows:

- (i) The channel access resolution is done via slotted CSMA/CA, where IEEE 802.15.4 beacon-enabled mode of communication is followed among the nodes, and all communications happen in contention access mode.
- (ii) New arrival process at a node is Poisson distributed with rate λ . The packets are of fixed size N slots. Thus, probability of a new arrival in a slot is $p = \lambda/N$.
- (iii) The communications are unacknowledged.
- (iv) Unlike in IEEE 802.15.4 standard, channel sampling is done over one slot, which is justified because of unacknowledged transmissions [10]. Each time a node wishes to transmit a data packet, it waits for a random number of slots. Following that backoff, if the channel is found idle for one carrier sense state (one slot), it transmits the data. If the channel is found busy after the random backoff, the node waits for another random period before trying to access the channel again.
- (v) Although in random deployment settings the actual number of forwarding neighbors (the number of contending nodes, or node density) may vary at different forwarding stages, for simplicity we assume it to be a constant M .
- (vi) The nodes do not have any buffer. Until the current packet is transmitted, the new arrivals are not accepted. Note that, for tractability of the analysis (as in [9]), finite buffer scenario is not considered.
- (vii) No data aggregation takes place at intermediate nodes.
- (viii) No forwarding priority is given between newly generated traffic at a node and the relay traffic it receives.

The above assumptions define the fundamental characteristics of the network and provide a basis for the analysis.

*For simplicity, without loss of generality of throughput analysis, the other likely states of a sensor, viz., idle listening and sleeping, are not considered for throughput modeling. All four states need to be considered when calculating the nodal energy consumption.

The assumptions on node distribution and data aggregation at intermediate stages can be relaxed in a more general analysis, however the current assumptions are sufficient to provide the intuition on many-to-one multihop throughput performance.

B. The throughput model

The system under consideration can be modeled as a collection of M to 1 clusters at each hop (Fig. 2). The nomenclature

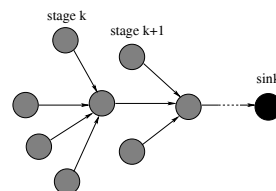


Fig. 2. The formation of M to 1 forwarding clusters in the network.

of the position of a node is done in terms of stages starting the nodes at the outermost region of the network (cluster) till the central sink. Data rate at the k -th stage is measured in terms of number of packet per packet length. A node at the k -th stage offers a data rate of λ_k to a node at the stage $k+1$. Hence, a node at stage 1 handles a data rate of $\lambda_1 = \lambda$. An intermediate node handles data of its own in addition to the data it receives from M nodes of the previous stage. Since the field nodes do not have a buffer space, a node may not always be able to forward all packets to the next stage.

The new traffic arrival rate being Poisson, with sufficiently large backoff window, the aggregated offered traffic at the k -th stage can be approximated as Poisson. Constant packet size, along with a random delay due to contention related backoff leads to a general distributed total service time. Recalling the no buffer assumption, the relation between λ_k and λ_{k+1} can be found by modeling a node at the stage $k+1$ as a two-state $M/G/1/1$ system (Fig.3). The states R and T are respectively

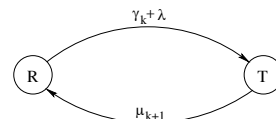


Fig. 3. Two-state system model of a relay node in stage $k+1$.

the receive and the transmit states of a relay node.

In the one-slot carrier sense model (assumption (iv)), the channel in a backoff slot can have three possible states: idle - I (i.e., no packet arrival); success - S (i.e., only one node transmits); or failure - F (i.e., more than one node transmit) – given that the previous slot was idle. In the k -th forwarding stage, denoting the idling-to-idling state transition probability as α_k and the idling-to-successful state transmission probability as β_k , the state transition relationships can be represented as in Fig. 4 [10]. Denote by π_{I_k} , π_{S_k} , and π_{F_k} , respectively, the idling, success, and failure state probabilities. Solving for π_{I_k} , π_{S_k} , and π_{F_k} , and noting that the state ‘I’ exists for one slot, and the states ‘S’ and ‘F’ exist for N slots (the entire

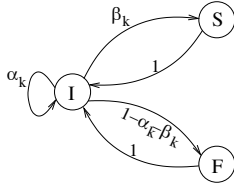


Fig. 4. Chanel state transition diagram in the k -th forwarding stage.

packet length), the channel throughput, i.e., the fraction of time spent in successful transmissions γ_k is obtained as:

$$\gamma_k = \frac{N\beta_k}{1 + (N-1)\alpha_k} \quad (1)$$

α_k and β_k can be expressed in terms of nodal transmission probability $p_{T_k}^n$ and the channel idling probability $p_{I_k}^c$ as [10]:

$$\alpha_k = \left(1 - \frac{p_{T_k}^n}{p_{I_k}^c}\right)^M \quad (2)$$

$$\beta_k = M \frac{p_{T_k}^n}{p_{I_k}^c} \left(1 - \frac{p_{T_k}^n}{p_{I_k}^c}\right)^{M-1} \quad (3)$$

where $\frac{p_{T_k}^n}{p_{I_k}^c} = p_{T_k|I_k}^n$ is the probability that a node transmits in the current slot given that the previous slot was idle. Using the nodal state transition representation in [10], these quantities can be obtained in terms of M and λ (see Appendix A). Thus, the rate of traffic offered to $k+1$ stage, $\gamma_{k+1} = \lambda + \gamma_k \triangleq \lambda_k$ is known as a function of M and λ .

From the state probabilities and sojourn times, the average time taken to transmit a packet in the $k+1$ stage is:

$$\begin{aligned} t_{d_{k+1}} &= \pi_{I_k} \times 1 + \pi_{S_k} \times N + \pi_{F_k} \times N \\ &= \frac{N(1 - \alpha_k) + 1}{2 - \alpha_k} \end{aligned} \quad (4)$$

Hence, the service rate $\mu_{k+1} = \frac{1}{t_{d_{k+1}}}$ is known.

With the traffic arrival rate and service rate at the k -th stage known, we can calculate the throughput at the stage $k+1$. Since the steady state system distribution of the $M/G/c/c$ loss system is the same as in $M/M/c/c$ loss system [12, Ch. 5], the system steady state probabilities of the $M/G/1/1$ are the same as in $M/M/1/1$ system. Therefore, the probability that the node is in receiving state R is given by

$$P_R = \frac{1}{1 + \lambda_k / \mu_{k+1}} \quad (5)$$

Hence, a node in the k -th stage is effectively able to forward data to the next stage at a rate

$$\lambda_{k+1} = \lambda_k P_R = \frac{\gamma_k + \lambda}{1 + (\gamma_k + \lambda) / \mu_{k+1}} \quad (6)$$

Using the above relationship, with the known new traffic generation rate λ , the traffic handled by a node at a particular stage can be found out by recursively solving (1), (5), and (6).

In the following, we take some numerical values to compute the throughput at various stages of data forwarding.

IV. NUMERICAL RESULTS AND DISCUSSION

For the numerical calculations, the packet size has been assumed $N = 10$ slots. Consistent with the backoff exponents in the IEEE 802.15.4 standard, the approximated geometrically distributed backoff window sizes have been chosen [13] with parameter p_i^n , for $i = 1$ to 5, as 0.2222, 0.1176, 0.0606, 0.0606, and 0.0606, respectively.

The variation of the throughput γ_{k+1} with data rate per node λ_k for $M = 10$ is shown in Fig. 5. At very low values of

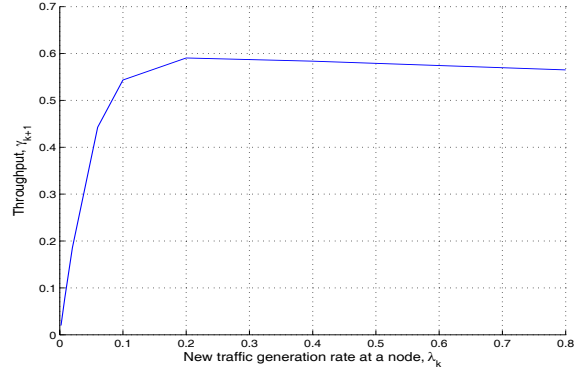


Fig. 5. Variation of throughput with new traffic arrival rate in a single M -to-1 cluster, with $M = 10$.

λ_k the collision probability between packets from M nodes is very less, and hence a majority of them are successfully transmitted. The throughput increases with λ_k until a stage arrives where the collisions become too frequent, leading to data loss. With further increase in λ_k the collisions increase, leading to the loss of throughput.

The variation of throughput with forwarding stages is shown in Fig. 6. When the new arrival rate λ is low, λ_k initially

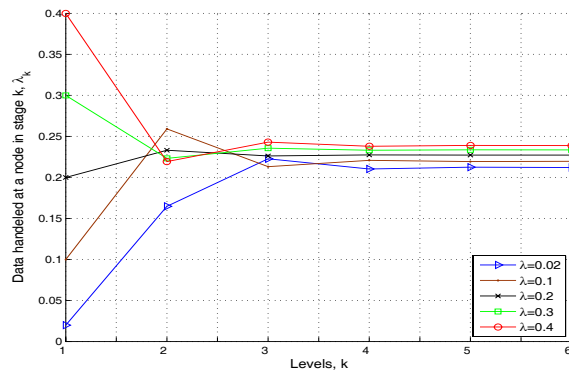


Fig. 6. Variation of λ_k with k , with node density $M = 10$.

increases with k . When collisions become large, it eventually stabilizes around a value $\lambda_s(\lambda)$ (saturation throughput) at a stage $k_s(\lambda)$ (saturation stage). With high λ , λ_k decreases before saturating, because the initial traffic is already too high, so only a fraction of newly generated packets are able to succeed in the contention process. Also, it is intuitive to see that the saturation stage reaches earlier with higher λ .

The dependence of λ_s on λ (as noted in Fig. 6) is shown clearly in Fig.7. As it can be seen in Fig.5, at high values of λ

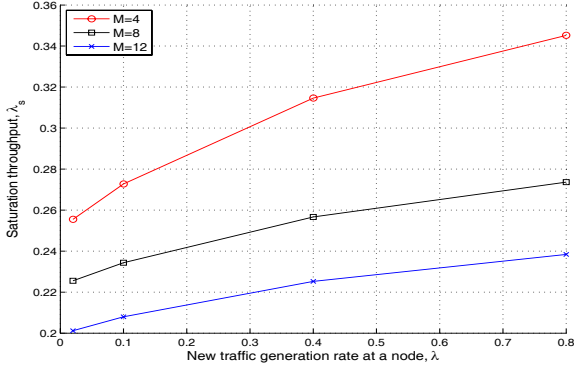


Fig. 7. Variation of λ_s with λ for different node densities.

the throughput of an M -to-1 cluster is almost flat. Hence if the nodes generate data at higher rate the effective data handled will be higher – which is reflected in higher λ_s and higher λ . It is also seen that at a higher value of M , λ_s is less, which is expected because of higher contention probability.

From Fig. 6 it was observed that, as the number of stages k increases, net data handled by the nodes initially increases and then saturates. To find the allowable number of stages (hops) to the sink, we define the *data accumulation gain* as the increase in volume of carried data from the previous stage. Fig. 8 shows that the *cluster size* (i.e., the maximum

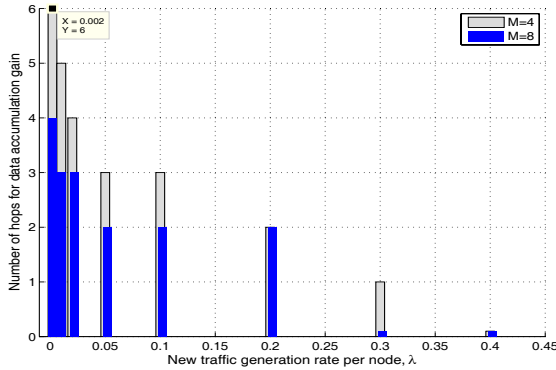


Fig. 8. Data accumulation gain versus hop count to the sink.

number of allowable hops to the sink) is highly dependent on the traffic generation rate and node density. Note that, even at a low traffic generation rate ($\lambda = 0.002$), the 4-to-1 forwarding cluster size is limited to only 6 hops. So, unless there is significant reduction in aggregated volume of data, the network (or cluster) size has to be quite limited for allowing the network to successfully carry (most of) the field data to the sink.

From the numerical studies it is observed how the three major parameters have to be decided while designing a multi-hop radial (many-to-one) network, which are: data generation

rate per node λ , node density M , and the radial depth of the network, i.e., the number of stages (hops) k to the central sink.

V. CONCLUDING REMARKS

We have analyzed the throughput performance characteristics of a radial (many-to-one) multihop wireless sensor network. Our numerical results showed that, for a given per node traffic generation rate, the throughput attains a saturation value beyond a limited number of hops to the sink. Conversely, the number of hops to be traversed dictates the maximum allowable traffic generation rate that can be forwarded to the sink without loss. These observations highlight the correlation between network traffic generation rate and the allowable distance to the sink. This observation will be useful in determining the energy consumption of different nodes, and hence the unpartitioned network's lifetime. The results can also be useful in devising network design and deployment strategies.

In our future work we will relax some of the constrained assumptions to obtain a more general conclusion. Network simulation will also be carried out to verify the analysis.

APPENDIX A

Determining $p_{I_k}^c$ and $p_{T_k}^n$ as a function of λ , M , and N :

This analysis is followed up from [10]. Refer to the nodal state transition diagram in Fig. A.1, where, for the sake of

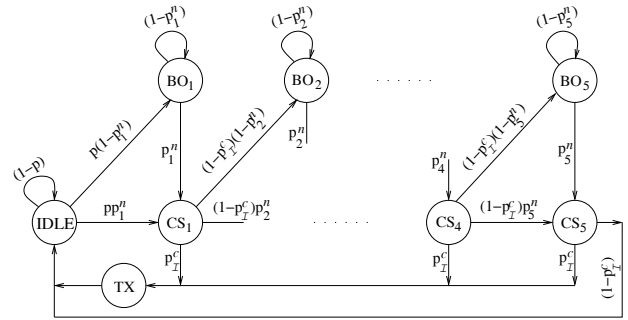


Fig. A.1. Imbedded Markov chain model of a transmitting sensor node with one-slot carrier sense (CS) state (from [10]). A node does maximum five random backoff (BO) to attempt a packet transmission.

notational simplicity, the subscript k referring to the k -th forwarding stage is omitted. The probability that the channel is idle is $p_{I_k}^c$. The number of random slots a node spends in BO_i is approximated as geometrically distributed with parameter p_i^n , such that the probability of transition out of BO_i in τ slots is $(1 - p_i^n)^\tau p_i^n$ [13]. The values of p_i^n are appropriately chosen to have a mean which is same as that of the uniform distributed backoff slots.

The state probabilities π_{I_k} (Idle state), $\pi_{B_{ki}}$ (i -th backoff state), $\pi_{C_{ki}}$ (i -th carrier sense state), and π_{T_k} (Transmit state) can be obtained after simple manipulation of the state equations:

$$\begin{aligned}
\pi_{B_{k1}} &= \frac{p_k}{p_1^n} (1 - p_1^n) \pi_{I_k}; & \pi_{C_{k1}} &= p_k \pi_{I_k} \\
\pi_{B_{2k}} &= \frac{p_k}{p_2^n} (1 - p_{I_k}^c) (1 - p_2^n) \pi_{I_k}; & \pi_{C_{2k}} &= p_k (1 - p_{I_k}^c)^2 \pi_{I_k} \\
\pi_{B_{3k}} &= \frac{p_k}{p_3^n} (1 - p_{I_k}^c)^2 (1 - p_3^n) \pi_{I_k}; & \pi_{C_{3k}} &= p_k (1 - p_{I_k}^c)^3 \pi_{I_k} \\
\pi_{B_{4k}} &= \frac{p_k}{p_4^n} (1 - p_{I_k}^c)^3 (1 - p_4^n) \pi_{I_k}; & \pi_{C_{4k}} &= p_k (1 - p_{I_k}^c)^4 \pi_{I_k} \\
\pi_{B_{5k}} &= \frac{p_k}{p_5^n} (1 - p_{I_k}^c)^4 (1 - p_5^n) \pi_{I_k}; & \pi_{C_{5k}} &= p_k (1 - p_{I_k}^c)^5 \pi_{I_k}
\end{aligned} \tag{A.1}$$

and

$$\pi_{T_k} = p_k \left[1 + (1 - p_{I_k}^c)^5 \right] \pi_{I_k}; \pi_{T_k} + \pi_{I_k} + \sum_{i=1}^5 (\pi_{B_{ki}} + \pi_{C_{ki}}) = 1 \tag{A.2}$$

where $p_k = \frac{\lambda_k}{N}$, the packet arrival probability in a slot.

Further, $p_{T_k}^n$ and $p_{I_k}^c$ are related as:

$$p_{I_k}^n = \left(\frac{\sum_{i=1}^5 \pi_{C_{ki}}}{\pi_{I_k} + N\pi_{T_k} + \sum_{i=1}^5 (\pi_{C_{ki}} + \pi_{B_{ki}})} \right) p_{I_k}^c \tag{A.3}$$

and from Fig. 4, going by the same argument as in throughput computation, we have the channel idling probability:

$$p_{I_k}^c = \frac{1}{1 + N(1 - \alpha_k)} \tag{A.4}$$

$p_{I_k}^c$ and $p_{T_k}^n$ are obtained in terms of λ , M , and N by numerically solving the intrinsic equations in (A.1)-(A.4).

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