Subcarrier Based Resource Allocation

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***Abstract*—Normally resource allocation is considered from the view point of users and the algorithms are designed so as to maximize total data rate supported while maintaining some constraints on quality of service and fairness. In this paper we approach the problem from the subcarrier's perspective. We propose a novel subcarrier based resource allocation scheme that assigns a subcarrier either to a single user or to more than one user on time shared basis according to the users' signal to noise ratios. This approach offers extra degree of freedom because unlike in user based schemes, in subcarrier based approach each user in principle can contend for any subcarrier unless it has already fulfilled its rate demand. Our simulation results demonstrate improved capacity without degradation of fairness.**

***Index Terms*—OFDMA, resource allocation, capacity, fairness**

I. INTRODUCTION

Orthogonal frequency division multiple access (OFDMA) has been widely used as a robust communication technique against the fading effects of wireless channel. OFDMA offers multiuser diversity as an extra degree of freedom along with frequency diversity. Resource allocation in OFDMA is a combination of subcarrier allocation, bit and power allocation over subcarriers. To support the growing bandwidth demand of the users, resource allocation strategies need to be highly sophisticated so as to maintain users' quality of service (QoS) demands and yet increase the network capacity with limited channel resource. Two classes of optimization techniques have been proposed for resource allocation namely, rate adaptive and margin adaptive. Rate adaptive techniques maximize the total data rate subject to transmission power constraint and margin adaptive techniques tries to minimize the total power given constraints on users' minimum supported data rates. Over the years rate adaptive techniques have gathered more interest among the research community due to their open objective of capacity maximization.

Rhee and Cioffi [1] proposed in a landmark paper in the category of rate adaptive algorithms that, each subcarrier should be assigned to a user that has a good channel gain over it. But, for fair allocation of resources, the algorithm initially allocates the best subcarrier to the set of users and then keep on picking the least served user and allocating the best subcarrier to the selected user. Maintaining fairness among users adds another constraint to the overall optimization problem making resource allocation an NP hard problem, so many suboptimal dynamic allocation strategies have been proposed [2]–[7]. Considering the load on the reverse channel for collecting channel state information (CSI) for each subcarrier per OFDM symbol, resource allocation is now considered as allocation of slots on the time-frequency map [8]–[10]. This approach is considered for WiMax systems and it helps in reducing the print of the downlink map [11].

We classify all these algorithms as user based resource allocation (UBRA) as these algorithms have been designed looking at resource allocation from users' perspective. The primary concern is maximization of total data rate and algorithm tries to find the best slot for a user in terms of average SNR. In order to maintain fairness algorithm picks least served user till all the resource are exhausted. We define another class of allocation as subcarrier based resource allocation (SBRA) where the ideology is to find the best user(s) for a subcarrier. For each subcarrier, SBRA algorithm finds the set of equally capable users based on received SNRs. Either there is a single user to own a subcarrier or there are multiple users to time share the subcarrier. Thus for each subcarrier we have fair contention among users which offers an extra degree of freedom to the SBRA that results in improved throughput and capacity.

Presenting a novel view of resource allocation from subcarrier's perspective allowing time sharing among equally capable users can be considered as the main contribution of this paper. Conceptual difference between UBRA and SBRA has been presented in Section II, which also describes the proposed SBRA. We present a proof of concept showing the improvement in capacity and fairness due to SBRA compared to UBRA in section

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III. The performance of the two algorithms in terms of OFDMA system throughput and maximum number of users supported subject to QoS constraints has been presented in Section IV and then section V concludes the paper.

II. CONVENTIONAL VERSUS PROPOSED RESOURCE ALLOCATION ALGORITHM

The algorithm presented in [1] is considered as the benchmark for the class of rate adaptive algorithms providing constrained fairness solution to the resource allocation problem and is considered as representative for the UBRA class. The slotted version of UBRA, which was proposed and studied in [10], is considered for performance comparison with respect to our proposed SBRA scheme.

A. User based resource allocation (UBRA)

This algorithm first allocates the best subcarrier according to maximum SNR to each user in a *for loop* and then keep on picking the least served user in a *while loop* and allocate the best available subcarrier to that user, until all the subcarriers are allocated [1]. The resulting fairness in this strategy is based on the basic assumption that the number of subcarriers is much larger compared to the number of users. In the slotted version of the algorithm, slots on the time-frequency plane are allocated to the users [10].

B. Subcarrier based resource allocation (SBRA)

In SBRA, we propose to time share the resources among equally capable users. Consider an OFDMA system where slots on frequency time plane are to shared among users. Let us assume one subcarrier per frequency slot and S OFDM symbols per time slot. Suppose over a subcarrier a user having maximum SNR is able to support a data rate R_{max} according to some discrete rate adaptation criterion. Then, if there are $(m-1)$ other users that too can support the same data rate on that subcarrier, then we have three options:

1) Allocate the subcarrier to the user having maximum SNR for the entire time slot. This scheme, termed as 'no sharing' scheme, has been a conventional approach proposed in [1], [10], [12], [13]. We consider it for comparison with our proposed approaches described below.
2) Time-share the subcarrier among m users by dividing OFDM symbols among them equally. This is one of the proposed schemes, which is called 'equal sharing' scheme. This approach is considered for theoretical discussion but it has practical limitations.
3) Time-share the subcarrier among, say, $\min(m, S)$ users by dividing OFDM symbols among them uniformly. This modified proposed scheme is identified as 'uniform sharing' scheme. This is a practical approach, which is also studied further in the paper.

Below, we state a proposition that provides the basis for improvement in fairness while retaining the capacity.

Proposition II.1. *The OFDMA systems implementing the three different schemes of subcarrier allocation, i.e., allocating a subcarrier to single best user, equally sharing the subcarrier among* m *users, and uniformly sharing the subcarrier among* $\min(m, S)$ *users, offer the same capacity.*

Proof of the above proposition is given in Appendix A.

This proposition asserts that, in all the three subcarrier allocation strategies, namely 'no sharing', 'equal sharing', and 'uniform sharing', the capacity is the same. However, allowing sharing of a subcarrier among more than one user adds another degree of freedom that may result in better short-term fairness. Thus, by increasing the granularity of resource allocation from time slot level to OFDM symbol level, i.e., by allowing sharing of the OFDM symbols within a slot among the competitive users, we may achieve a higher short-term fairness without any loss of capacity. Based on this concept, below we outline our proposed 'uniform sharing' algorithm that works at the level of supported data rates.

The above algorithm can be easily extended for allocation of slots on the frequency-time plane in OFDMA systems. In that case, each slot is considered independently for allocation on time-shared basis among the competitive users. We consider the slotted version of SBRA for comparative performance study with the slotted UBRA [10] in section IV.

III. TWO USER TWO SUBCARRIER SCENARIO

In this Section we characterize the capacity and fairness performance of UBRA and the proposed SBRA schemes. The channel is considered to experience frequency selective Rayleigh fading. For simplicity of capturing the basic performance gain, capacity of the system is measured in terms of bits per subcarrier and fairness is measured by Jain's fairness index [14].

Let us denote the received SNR matrix as:

$$\underline{\mathbf{\Gamma}} = \begin{pmatrix} \gamma_1^1 & \gamma_2^1 \\ \gamma_1^2 & \gamma_2^2 \end{pmatrix}$$

where γ_j^i is the SNR of user-i on jth subcarrier. In

Rayleigh fading channel with average received SNR $\overline{\gamma}$, the SNR $\boldsymbol{\gamma}$ is exponentially distributed with probability density function given by:

$$\mathrm{f}_{\boldsymbol{\gamma}}(\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}$$

For the time being, let us assume two level adaptive modulation scheme, i.e., if the received SNR is above a threshold γ_{th}, the supported data rate is b, otherwise it is 0.

1) UBRA: Since the number of users is equal to the number of subcarriers, the resource allocation to all the users is done in one go, as outlined in Section II-A. In this case, the algorithm compares γ_1^1 and γ_2^1. If $\gamma_1^1 > \gamma_2^1$, the allocation matrix is

$$\underline{\boldsymbol{A}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

which means user-1 uses subcarrier-1. Otherwise

$$\underline{\boldsymbol{A}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Note that user-2 has no other option but to go with the leftover subcarrier.

Let us analyze the case when $\gamma_1^1 > \gamma_2^1$. There are three possible sub-cases: $(i)\, \gamma_{th} > \gamma_1^1$; $(ii)\, \gamma_1^1 > \gamma_{th} > \gamma_2^1$; $(iii)\, \gamma_2^1 > \gamma_{th}$. User-1 can use subcarrier-1 in case (ii) and (iii) only. Hence the capacity assigned to user-1 is given by: $C_{u_1} = b(1 - \frac{1}{2}e^{-\beta})e^{-\beta}$, where $\beta = \frac{\gamma_{th}}{\overline{\gamma}}$. Correspondingly, the capacity assigned to user-2 on subcarrier-2 is $C_{u_2} = \frac{b}{2}e^{-\beta}$.

Capacity assigned to both the users in the other case (when $\gamma_1^1 < \gamma_2^1$) is the same as above. Thus, the overall capacities of the two users are $C_{u_1} = 2b\left(1 - \frac{1}{2}e^{-\beta}\right)e^{-\beta}$ and $C_{u_2} = be^{-\beta}$.

Accordingly, the total capacity of the UBRA scheme is obtained as:

$$\begin{aligned} \mathrm{C}_{ubra} &= 2b\left(1 - \frac{1}{2}e^{-\beta}\right)e^{-\beta} + be^{-\beta} \\ &= b\left(1 - \left(1 - e^{-\beta}\right)^2\right) + b\left(1 - \left(1 - e^{-\beta}\right)\right) \end{aligned} \tag{1}$$

2) SBRA: As outlined in Section II-B, in SBRA a subcarrier is assigned to either one user or shared among more than one user depending on the maximum rate offered by the subcarrier to the users. Since the capacity of all the three schemes of SBRA namely 'no sharing', 'equal sharing', and 'uniform sharing' is the same (see proposition II.1), we present the analysis of the 'equal sharing' scheme only. The algorithm works on individual subcarriers. For example on subcarrier-1 γ_1^1 and γ_1^2 are compared with the γ_{th}. There exist four cases:

(i) $\gamma_1^1 > \gamma_{th}$ and $\gamma_1^2 > \gamma_{th}$: In this case subcarrier has to be time-shared between the two users. Considering equal sharing, their respective capacities are $C_{u_1} = C_{u_2} = \frac{b}{2}e^{-2\beta}$.

(ii) $\gamma_1^1 > \gamma_{th}$ and $\gamma_1^2 < \gamma_{th}$: In this case the subcarrier is used by user-1 alone and the capacity assigned is $C_{u_1} = b\left(1 - e^{-\beta}\right)e^{-\beta}$.

(iii) $\gamma_1^1 < \gamma_{th}$ and $\gamma_1^2 > \gamma_{th}$: In this case the subcarrier is used by user-2 and the capacity assigned is $C_{u_2} = b\left(1 - e^{-\beta}\right)e^{-\beta}$.

(iv) $\gamma_1^1 < \gamma_{th}$ and $\gamma_1^2 < \gamma_{th}$: In this case the subcarrier is not usable by any of the users.

Thus, each user's capacity over a single subcarrier is given by: $C_{u_1} = \mathrm{C}_{u_2} = b\left(1 - \frac{1}{2}e^{-\beta}\right)e^{-\beta}$.

Since all subcarriers are independent, the overall capacity in SBRA scheme is obtained as:

$$\mathrm{C}_{sbra} = 2b\left(1 - \left(1 - e^{-\beta}\right)^2\right) \tag{2}$$

3) Capacity improvement of SBRA over UBRA: The achieved capacity gain in SBRA over UBRA can be obtained as:

$$\mathrm{C}_{gain} = C_{sbra} - C_{ubra} = b\left(1 - e^{-\beta}\right)e^{-\beta} \tag{3}$$

which has a peak at average SNR

$$\overline{\gamma}_{\mathbf{max}} = \frac{\gamma_{th}}{\ln(2)} \tag{4}$$

As observed from (1) and (2), assigned capacities to the users are different in UBRA, while they are the same in SBRA, thereby indicating improved fairness. Further, accounting the gain, this simple case of 2 users 2 subcarriers demonstrates the capacity gain and fairness improvement by SBRA.

IV. SIMULATION RESULTS AND DISCUSSION

The algorithm proposed in [10] has been considered as the UBRA representative algorithm for the comparison of our algorithm. We have adapted our algorithm accordingly for allocation of slots on time-frequency map. Both these algorithms have been compared for throughput as well as maximum numbers of users supported (capacity) under various traffic cases.

A. OFDMA System Model

We assume an OFDMA system based on IEEE 802.16e where K users are supported by the base station (BS). Carriers are divided in bins, with each bin having 8 data subcarriers and 1 pilot subcarrier. We assume a slotted structure on time-frequency plane with N = 2 (consecutive bins of subcarriers in slot) and M = 3 (consecutive OFDM symbols) according to the IEEE

802.16e standard for the AMC permutation [10]. Users are allocated these slots on the time-frequency plane.

We assume that the frame is used in TDD mode with the downlink subframe followed by the uplink subframe. There is a gap of 5 microseconds between the subframes and also between two consecutive frames. The frame duration is assumed to be 8ms and there are 40 OFDM symbols on downlink frame and 39 OFDM symbols in uplink frame. The OFDMA system parameters are tabulated in table I.

TABLE I
OFDMA SYSTEM PARAMETERS

Carrier frequency	3.5 GHz
Total Bandwidth	10 MHz
Number of Subcarriers	1024
Frame duration	8 ms
Symbol duration	100 μs
Subcarriers in a slot	18
OFDM symbols in a slot	3
Modulation scheme	QPSK, 16 & 64 QAM

Perfect Channel State Information (CSI) is assumed at the transmitter and the BS uses this information to allocate slots to different users based on their requirements. Also we assume that the channel shows quasi-static nature so that the allocation done is valid for the entire duration of the frame.

Depending on the channel state the transmitter uses either of the QPSK, 16-QAM and 64 QAM modulation scheme. If the received SNR is below certain threshold then that subcarrier is not used by any of the users. We have assumed ITU-R vehicular channel model A with 6 paths for our simulation. Maximum Doppler deviation is f_d = 408Hz and the maximum delay spread is $\tau_{max} = 2.51\mu s$.

B. Throughput Improvement

We assume that there are total 20 users in the OFDMA system. All the users have infinite traffic in their queues and they shall always be using the slots offered to them. This scenario gives us an idea about the total throughput that the system can offer. Figure 1 shows the percentage improvements in the throughput offered by our algorithm in comparison to the algorithm suggested in [10]

The proposed algorithm gives a maximum gain of around 6% compared to the proportional fair algorithm suggested in [10]. The percentage gain reduces with SNR as both the algorithms tries to achieve the maximum achievable throughput at higher SNRs.

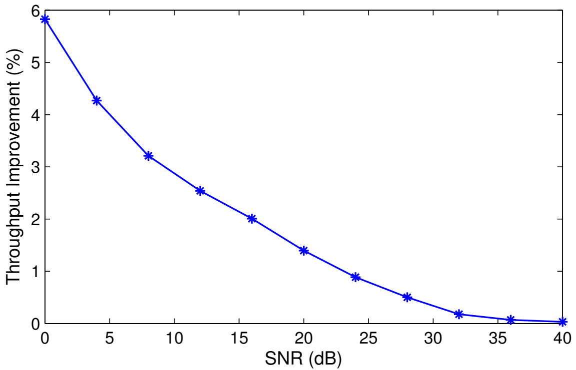

Fig. 1. Percentage throughput gain for infinite traffic case.

C. Maximum number of CBR users supported under QoS

In order to have a fair comparison between the two algorithms we find the maximum numbers of CBR traffic users that can be supported for a fixed packet drop rate of 1%. If a packet can not be delivered within time limit the packet shall be dropped. Table II summarizes the parameters used for the simulation of CBR and VBR traffic.

TABLE II
PARAMETERS USED FOR TRAFFIC GENERATION

CBR	
Packet size	70 Bytes
Delay bound	40 ms
Packet drop rate	1 percent
VBR	
Mean ON period	1.47 s
Mean OFF period	1.92 s
Packet inter-arrival time	5 ms
Packet size	70 to 1500 Bytes
Delay bound	200 ms
Packet drop rate	1 percent

Maximum numbers of users supported for a constrained packet drop rate gives an indication of the capacity of the system. Our proposed algorithm performs better then the proportional fair algorithm and the gain shown by SBRA is because of the extra degree of freedom.

D. Maximum number of VBR users supported under QoS

In this case users are assumed to have a VBR traffic that follows ON-OFF model. Maximum numbers of users supported by the algorithm is compared for a fixed packet drop rate of 1 percent. The delay tolerance for this case is 200 ms.

In case of VBR the improvement in numbers of users supported is significant and grows exponentially with

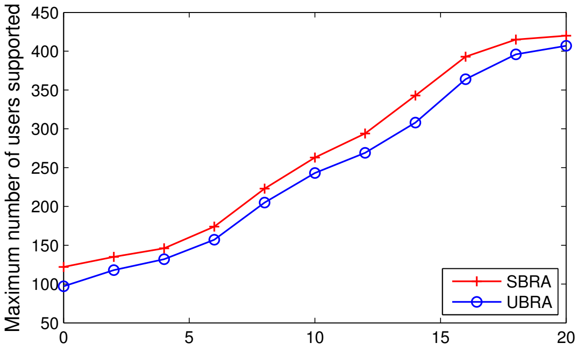

Fig. 2. Maximum number of users supported for CBR traffic.

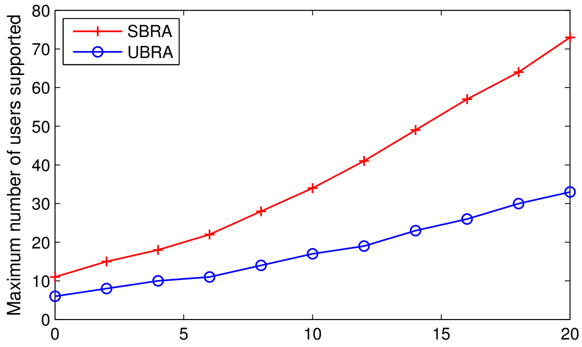

Fig. 3. Maximum number of users supported for VBR traffic.

SNR. Here the gain in SBRA is two fold, one is the fixed gain and another is variable gain due to the adaptivity of SBRA compared to UBRA with respect to user's demands.

V. CONCLUSION

In this paper we presented a novel perspective of looking at the resource allocation problem in OFDMA systems. We presented a proof of concept to state that there is an improvement in throughput by considering the problem from the subcarrier perspective rather than user perspective. Our simulation results approves our idea of improvement in throughput and maximum numbers of users supported for the CBR and VBR traffic case.

APPENDIX A
PROOF OF PROPOSITION: CAPACITY ANALYSIS OF 'NO SHARING', 'EQUAL SHARING', AND 'UNIFORM SHARING' SCHEME

Proof: Let us assume that there are M users contending for a single subcarrier. Also consider two level adaptive modulation scheme where, if the received SNR is greater than a SNR threshold γ_{th} then the subcarrier can support b bits, otherwise the subcarrier is not used at all.

A. *Capacity of 'no sharing' allocation scheme*

Since all users are assumed to have the same average received SNR, probability that user-i has the maximum SNR is $\frac{1}{M}$. Probability that the maximum SNR is above threshold is given by $P = 1 - \left(1 - e^{-\beta}\right)^M$. Correspondingly, the capacity assigned to each user is $C_{u_i} = \frac{b}{M}\left(1 - \left(1 - e^{-\beta}\right)^M\right)$ and the overall capacity of 'no sharing' scheme is obtained as:

$$\mathrm{C}_{nos} = b\left(1 - \left(1 - e^{-\beta}\right)^M\right) \qquad \text{(A.1)}$$

B. *Capacity of 'equal sharing' allocation scheme*

Probability that m users' SNRs are above threshold is: $P = C_m^M \left(e^{-\beta}\right)^m \left(1 - e^{-\beta}\right)^{M-m}$. When resources are shared equally, the capacity assigned to each one of the users in the set of m users is given by: $C_{u_i} = \frac{b}{m} C_m^M \left(e^{-\beta}\right)^m \left(1 - e^{-\beta}\right)^{M-m}$. Probability of any user getting selected in the above m users' group is $\frac{m}{M}$. Hence, average capacity assigned to each one of the users in the set of M users is obtained as: $\mathrm{C}_{u_i} = \frac{b}{M} C_m^M \left(e^{-\beta}\right)^m \left(1 - e^{-\beta}\right)^{M-m}$. The total capacity of each user is obtained by averaging over all the possible values of m:

$$\begin{aligned}\mathrm{C}_{u_i} &= \sum_{m=1}^{M} \frac{b}{M} C_m^M \left(e^{-\beta}\right)^m \left(1 - \left(e^{-\beta}\right)\right)^{M-m} \\ &= \frac{b}{M}\left(1 - \left(1 - e^{-\beta}\right)^M\right)\end{aligned}$$

Thus, the overall capacity of the 'equal sharing' scheme is

$$\mathrm{C}_{eqs} = b\left(1 - \left(1 - e^{-\beta}\right)^M\right) \qquad \text{(A.2)}$$

C. *Capacity of 'uniform sharing' scheme*

Let us assume there are $S(< M)$ OFDM symbols in a slot, that are to be shared uniformly among the users. Probability that there are m users having their SNRs above threshold is given by: $P = C_m^M \left(e^{-\beta}\right)^m \left(1 - \left(e^{-\beta}\right)\right)^{M-m}$.

Till $m \leq S$, the capacity assigned to each user is: $C_{u_i} = \frac{b}{M} C_m^M \left(e^{-\beta}\right)^m \left(1 - \left(e^{-\beta}\right)\right)^{M-m}$. For $m > S$, the contending users are more in number compared to the available OFDM symbols, hence available resources are to be shared among S users by a factor of $\frac{1}{S}$. Probability of picking S users uniformly out of m users is $\frac{S}{m}$ and probability of picking m users, having the same high SNR, among the total M users is $\frac{m}{M}$. Thus, the capacity of each user is given by: $C_{u_i} = \frac{b}{M} C_m^M \left(e^{-\beta}\right)^m \left(1 - \left(e^{-\beta}\right)\right)^{M-m}$ which is the same as the capacity assigned when $m \leq S$. Thus, the total capacity of each user obtained by averaging over all possible values of m is given by:

$$\mathrm{C}_{u_i} = \frac{b}{M}\left(1 - \left(1 - e^{-\beta}\right)^M\right)$$

and the overall capacity of the 'uniform sharing' scheme is

$$\mathrm{C}_{ufs} = b\left(1 - \left(1 - e^{-\beta}\right)^M\right) \tag{A.3}$$

Since the capacity assigned on a single subcarrier is same for all the three schemes, the overall OFDMA capacity assigned by all the three schemes will be the same. □

REFERENCES

[1] W. Rhee and J. M. Cioffi, "Increase in capacity of multiuser OFDM system using dynamic subchannel allocation," in *Proc. IEEE VTC 2000-Spring*, vol. 2, Tokyo, Japan, May 2000, pp. 1085–89.

[2] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2726–36, Nov 2005.

[3] I. C. Wong, Z. Shen, B. L. Evans, and J. G. Andrews, "A low complexity algorithm for proportional resource allocation in OFDMA systems," in *Proc. IEEE Wksp. Sig. Proc. Sys. Design and Implementation*, Austin, TX, Oct 2004, pp. 1–6.

[4] H. Kim and Y. Han, "A proportional fair scheduling for multicarrier transmission systems," *IEEE Comm. Lett.*, vol. 9, pp. 210–12, 2005.

[5] Y. Ma, "Rate maximization for downlink OFDMA systems," *IEEE Trans. Vehicular Technol.*, vol. 57, no. 5, pp. 3267–74, Sep 2008.

[6] C. Mohanram and S. Bhashyam, "A sub-optimal joint subcarrier and power allocation algorithm for multiuser OFDM," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 685–87, Aug 2005.

[7] N. Y. Ermolova and B. Makarevitch, "Low complexity adaptive power and subcarrier allocation for ofdma," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 433–37, 2007.

[8] Y. Ben-Shimol, I. Kitroser, and Y. Dinitz, "Two-dimensional mapping for wireless OFDMA system," *IEEE Trans. Broadcasting*, vol. 32, no. 3, pp. 388–96, Sep 2006.

[9] T. Wang, H. Feng, and B. Hu, "Two-dimensional resource allocation for OFDMA system," in *Proc. IEEE International Conference on Communications Workshops*, May 2008, pp. 1–5.

[10] A. Biagioni, R. Fantacci, D. Marabissi, and D. Tarchi, "Adaptive subcarrier allocation schemes for wireless OFDMA systems in WiMAX networks," *IEEE J. Sel. Areas in Commun.*, vol. 27, no. 2, pp. 217–25, Feb 2009.

[11] *Standard for local and metropolitan area networks Part 16: Air interface for fixed and mobile broadband wireless access systems, IEEE Std. 802.16e*, 2005.

[12] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *IEEE J. Sel. Areas in Commun.*, vol. 17, no. 10, pp. 1747–58, Oct 1999.

[13] J. Jang and K. B. Lee, "Transmit power adaptation for multiuser OFDM systems," *IEEE J. Sel. Areas in Commun.*, vol. 21, no. 2, pp. 171–78, Feb 2003.

[14] R. Jain, D.-M. Chiu, and W. R. Hawe, *A quantitative measure of fairness and discrimination for resource allocation in shared computer system*. Eastern Res. Lab., Digital Equipment Corp., 1984.