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Utility-Fair Wireless Resource Allocation for Heterogeneous Users

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***Abstract*—To move towards low-latency wireless communication while taking care of elastic traffic, proper resource allocation is very important to provide acceptable quality of service (QoS) to all users. This work solve two optimization problems which aim at utility-proportional fairness maximization by optimizing power and bandwidth (BW) allocation among heterogeneous users having real-time and elastic traffic. Noting that both problems are non-convex, local optimal solutions in terms of BW and power are found by alternating optimization algorithm. Numerical results validate analysis, represent schedulability and average utility of real-time traffic user with respect to available resources, and compare utility fairness performance of both schemes.**

***Keywords*—Heterogeneous users, power control, BW allocation, schedulability, utility-proportional fairness, alternating optimization**

I. Introduction

In the era of 5G with the goal of low-latency communication [1], Internet of Things (IoT), and multi-service communication, users have different types of resource demands, namely for delay-constrained real-time traffic (RT), e.g. video streaming, and delay-tolerant elastic traffic (ET), e.g. FTP. Due to heterogeneous service demands, where users need different quality of service (QoS), optimal resource allocation to maximize users satisfaction level and provide fairness among them is a challenging problem.

A. Related Works

As prior work, the problem of joint utility-based customized price and power control in multi-service wireless networks is addressed in [2] by a two variable non-cooperative game with respect to user's price and uplink transmission power. Joint bandwidth (BW) and power allocation is investigated in [3] to improve power efficiency with QoS guarantee. However, only RT users are considered. It is shown in [4] that, when network utility maximization (NUM) runs for only ET, the resulting resources allocated by the algorithm turn out to be equally divided to all its users. If RT is considered together with ET, NUM becomes a non-convex problem [5] due to a minimum rate required by RT, where an approximate optimal rate allocation is found. The results however may lead to unfair use of resource because ET users having logarithmic utility function are always given preference over RT users having sigmoidal utility function particularly at lower rates. To address fairness, utility-proportional fairness (UPF) is proposed in [6], [7] for wired networks with fixed link capacities, which is not applicable in wireless context. For wireless networks, a distributed rate allocation algorithm [8], [9] and rate and power allocation algorithm [10] are investigated to achieve UPF. In particular, a high signal-to-interference noise ratio (SINR) region is considered [10], while channel gain does not change over time. In contrast to [8]–[10], we consider joint optimal BW and power allocation in place of rate allocation, *while channel gain changes over time*.

In this paper we consider two most widely used definitions for the rate allocation to provide utility-proportional fairness to the users with heterogeneous traffic demands. In the wireless network since the rate capacity is not fixed due to variable channel conditions, we use these definitions for the power and bandwidth allocation in place of rate allocation while taking care of channel condition such that minimum required rate can be provided to RT users with tolerable loss, and utility fairness can be maintained. Also we want to check which approach provides better results in terms of serving higher number of RT users and provide better utility-fairness among heterogeneous users with fixed available power and bandwidth resources.

B. Key Contributions and Paper Organization

Key contribution of this work is to design two resource allocation algorithms to maximize UPF among heterogeneous types of traffic with respect to BW and power; one aims at maximizing sum of transformed utility function and the other provides nonzero rate by maximizing product of utility functions. Main challenges are that a minimum required rate of RT users can not be provided when channel gain is too low, while both UPF optimization problems are non-concave in joint BW and power allocations. To cope with these challenges, we consider a schedulability of the proposed algorithms such that violations of a minimum rate for RT users can be limited up to a tolerable level, which is acceptable for RT. We then develop two BW and power allocating algorithms to find local solutions for both optimization problems based on alternating optimization technique [11]. Numerical simulations represents the schedulability and average utility that the proposed algorithms provide to RT users with respect to available resources, and compare the performance of one algorithm over the other.

The rest of the paper is organized as follows: Section II represents the system model and utility functions used for the RT and ET users. In Section III, two optimization problems are posed based on two UPF definitions, and two algorithms are designed to solve these problems. Sections IV represents the numerical results and Section V concludes the paper.

II. System Model

We consider downlink scenario in a circular cell of radius R, where a BS is centered and N users are uniformly located. Among N users' traffic, we consider two types of traffic such as RT and ET. Let us denote by $\overline{N}_R$ and $\overline{N}_E$ the index set of RT and ET users, respectively. Additionally, $\overline{N}$ denotes the set of all the users such that $\overline{N} = \overline{N}_R \bigcup \overline{N}_E$. The cardinality

of the sets of RT and ET users is respectively denoted by $N_R = |\overline{N}_R|$ and $N_E = |\overline{N}_E|$ such that $N = N_R + N_E$.

In order to capture heterogeneous service demands, if user i has RT traffic, i.e., $i \in \overline{N}_R$, its utility function with rate r_i is a sigmoidal function as

$$U_i(r_i) = 1/(1 + e^{-a(r_i - b)}), \tag{1}$$

where a and b are positive constants and are known as calibration parameters of the sigmoidal utility. If its traffic is ET, i.e., $i \in \overline{N}_E$, then $U_i(r_i)$ takes form

$$U_i(r_i) = \log(1 + Kr_i)/\log(1 + Kr_{\max}), \tag{2}$$

where K is the rate of increase of utility percentage with the allocated data rate r_i, and $r_{\max}$ is the data rate for the user having ET to achieve maximum utility percentage i.e. 100%. Notice that (1) is non-convex, while (2) is concave.

We assume that the BS and users are equipped with a single antenna. The system time is slotted into a constant size. Each slot consists of uplink and downlink subslots, i.e., time-division duplex (TDD), whereas the system has a total of bandwidth B and a total power budget P_t. Over the uplink each user can report its channel gain obtained by making use of the reference signal of the downlink. Due to TDD, where channel gain may remain constant over a slot, the BS allocates a power P_i and a normalized fraction of BW for user i, denoted by η_i, in reference to the channel gain feedback in providing rate r_i. We assume that the channel undergo statistically independent frequency non-selective Rayleigh fading; that is, the channel gain h_i for user i is exponentially distributed with unit mean and a distance from the BS to user i is d_i. Then, rate r_i allocated to user i is expressed as

$$r_i \triangleq \eta_i B \log_2\left(1 + P_i h_i/({d_i}^{\alpha}\sigma^2)\right), \tag{3}$$

where $\eta_i \in [0, 1]$, α and σ^2 denote a pathloss and noise power.

III. Algorithm Design

In this section first we present the two definitions for UPF and pose two optimization problems based on these definitions. Then we design two algorithms to solve these optimization problem with respect to power and BW allocation. Schedulability of RT users is also defined in this section.

Definition 1: As defined in [6], a rate allocation vector $\boldsymbol{r}^* = [r_i^*]$ of length N is said to be utility-proportional fair if it is feasible and for any other feasible allocation $\boldsymbol{r} = [r_i]$,

$$\sum_{i \in \overline{N}} \frac{(r_i - r_i^*)}{U_i(r_i^*)} \leq 0. \tag{4}$$

In addition, UPF can be achieved if the utility $U_i(r_i)$ of user i is transformed according to

$$\mathcal{U}_i(r_i) = \int_{r_{m,i}}^{r_i} \frac{1}{U_i(y)} dy \text{ for } r_{m,i} \leq r_i, \tag{5}$$

where $r_{m,i}$ is the minimum rate for user i. We consider that $r_{m,i}$ for ET is zero and they are served with best-effort rates, while for a user with RT traffic, $r_{m,i} = r_m$ for $i \in \overline{N}_R$.

Definition 2: As defined in [8], [9], in order to provide non zero rates and a minimum QoS to the users having heterogeneous traffic, the product of utilities formulates UPF optimization objective function, which is expressed as follows:

$$\prod_{i \in \bar{N}} U_i(r_i) \tag{6}$$

By using these two definitions, we formulate to UPF optimization problems in next subsection.

A. Problem Formulation and Algorithm Design

According to (3), in realizing r_i, let us define two vectors of length N as $\boldsymbol{P} = [P_i]$ and $\boldsymbol{\eta} = [\eta_i]$, i.e., an allocated power and bandwidth, respectively. Now, two UPF maximization problems $P1$ and $P2$ are cast into joint optimization of power and BW allocation while keeping the same BW and power budgets as constraints for both problems. Using definition 1, the UPF optimization problem $P1$ is posed as:

$$P1: \underset{\boldsymbol{P},\boldsymbol{\eta}}{\text{maximize}} \sum_{i=1}^{N} \mathcal{U}_i\left(\eta_i B \log_2\left(1 + \frac{P_i h_i}{d_i^{\alpha}\sigma^2}\right)\right)$$

$$\text{subject to} \quad \sum_{i=1}^{N} \eta_i = 1, \tag{7a}$$

$$\sum_{i=1}^{N} P_i = P_t, \tag{7b}$$

where $\eta_i \geq 0$ and $P_i \geq 0$ for $i \in \overline{N}$.

Since the UPF objective function $\underset{\boldsymbol{P},\boldsymbol{\eta}}{\text{maximize}} \prod_{i \in \bar{N}} (U_i(r_i)$ in definition 2, is equivalent to $\underset{\boldsymbol{P},\boldsymbol{\eta}}{\text{maximize}} \sum_{i=1}^{N} \log(U_i(r_i)$, the UPF optimization Problem $P2$ is formulated as follows:

$$P2: \underset{\boldsymbol{P},\boldsymbol{\eta}}{\text{maximize}} \sum_{i=1}^{N} \log\left(U_i\left(\eta_i B \log_2\left(1 + \frac{P_i h_i}{d_i^{\alpha}\sigma^2}\right)\right)\right)$$

$$\text{subject to} \quad \sum_{i=1}^{N} \eta_i = 1, \tag{8a}$$

$$\sum_{i=1}^{N} P_i = P_t, \tag{8b}$$

where $\eta_i \geq 0$ and $P_i \geq 0$ for $i \in \overline{N}$.

Two important remarks should be made: Firstly, the two problems $P1$ and $P2$ are not concave problems in $\boldsymbol{P}$ and $\boldsymbol{\eta}$ as shown in Appendix. We therefore seek for a local maximizer and characterize a local optimal solution later. Secondly, the minimum rate r_m for RT users can not be always guaranteed due to the fact that channel gain $h_i d_i^{-\alpha}$ can take often too a low value probabilistically so that even if P_t is allocated to a single user, it can not be large enough to provide r_m to him. Therefore, we shall allow a reasonably small loss, i.e., the event that rate r_i is less than r_m, while the number of RT users should be limited to keep such losses small. To address this loss we define schedulability in next subsection.

To find a solution of $P1$, let us write Lagrangian of $P1$ with constraints in (7a), (7b) and by keeping the boundary constraints ($\eta_i \geq 0$ and $P_i \geq 0$ for $i \in \overline{N}$) implicit as:

$$\mathcal{L}_1 = \sum_{i=1}^{N} \mathcal{U}_i(r_i) - \lambda\left(\sum_{i=1}^{N} P_i - P_t\right) - \mu\left(\sum_{i=1}^{N} \eta_i - 1\right) \tag{9}$$

where λ and μ are Lagrangian multipliers. The Karush-Kuhn-Tucker (KKT) conditions are expressed as

$$\frac{\partial \mathcal{L}_1}{\partial \eta_i} = \frac{\partial \mathcal{U}_i(r_i)}{\partial r_i}\frac{\partial r_i}{\partial \eta_i} = \frac{B}{U_i(r_i)} \log_2\left(1 + \frac{P_i h_i}{{d_i}^{\alpha}\sigma^2}\right) - \mu = 0, \tag{10}$$

Algorithm 1 UPF Power and BW allocation algorithm for $P1$

1: At each time slot, do the following with $k = 0$, $P_i^{[k]} = P_t/N$ and $\eta_i^{[k]} = B/N$
2: **while** (1) **do**
3: with $P_i^{[k]}$, find μ, $\eta_i^{[k+1]}$ from (13), (14), (15).
4: with $P_i^{[k]}$, $\eta_i^{[k+1]}$, find λ and $P_i^{[k+1]}$ from (11), (7b).
5: **if** $\|\boldsymbol{P}^{[k+1]} - \boldsymbol{P}^{[k]}\| < \epsilon$ and $\|\boldsymbol{\eta}^{[k+1]} - \boldsymbol{\eta}^{[k]}\| < \epsilon$ **then**
6: break;
7: **else**
8: $k = k + 1$
9: **end if**
10: **end while**

$$\frac{\partial \mathcal{L}_1}{\partial P_i} = \frac{1}{U(r_i)} \frac{\eta_i B h_i}{\ln(2)(P_i h_i + {d_i}^{\alpha}\sigma^2)} - \lambda = 0, \ \forall i \in \bar{N} \quad (11)$$

where we have used $\frac{\partial \mathcal{L}_1}{\partial P_i} = \frac{\partial \mathcal{U}_i(r_i)}{\partial r_i}\frac{\partial r_i}{\partial P_i}$.

Let us introduce a numerical algorithm to find a local optimal solution, presented in Algorithm 1. From (10), we have

$$\eta_i = \frac{U_i^{-1}\left(\frac{B}{\mu}\log_2\left(1 + \frac{P_i h_i}{{d_i}^{\alpha}\sigma^2}\right)\right)}{B\log_2\left(1 + \frac{P_i h_i}{{d_i}^{\alpha}\sigma^2}\right)}. \quad (12)$$

Let η_i° and $\eta_i^{\bullet}$ be the solution of (12) for RT and ET, i.e., KKT point, respectively. Plugging (1) into (12), we get η_i° for RT users as

$$\eta_i^{\circ} = \frac{-\log\left(\mu/[B\log_2\left(1 + P_i h_i/({d_i}^{\alpha}\sigma^2)\right)] - 1\right) + ab}{aB\log_2\left(1 + P_i h_i/({d_i}^{\alpha}\sigma^2)\right)}, \quad (13)$$

whereas substituting (2) into (12) yields $\eta_i^{\bullet}$ for ET users as

$$\eta_i^{\bullet} = \frac{\exp\left(\left(\frac{B}{\mu}\right)\log_2\left(1 + \frac{P_i h_i}{{d_i}^{\alpha}\sigma^2}\right)\log(1 + Kr_{\max})\right) - 1}{KB\log_2\left(1 + \frac{P_i h_i}{{d_i}^{\alpha}\sigma^2}\right)}. \quad (14)$$

Since η_i° and $\eta_i^{\bullet}$ are a function of μ, we can find μ numerically using (7a) as

$$\sum_{i \in \overline{N}_R} \eta_i^{\circ} + \sum_{i \in \overline{N}_E} \eta_i^{\bullet} = 1. \quad (15)$$

This corresponds to the third line of Algorithm 1.

Let P_i^* for $i \in \overline{N}$ denote a solution of optimization problem $P1$. In order to find P_i^*'s and λ in (11), which are $N + 1$ unknown variables, we numerically solve (11) together with (7b), which are N nonlinear equations and one linear equation. This is given at the fourth step of Algorithm 1.

Now, to find a solution of optimization problem $P2$, let us write Lagrangian of $P2$ with constraints in (8a), (8b) and by keeping the boundary constraints ($\eta_i \geq 0$ and $P_i \geq 0$ for $i \in \overline{N}$) implicit as:

$$\mathcal{L}_2 = \sum_{i=1}^{N} \log(U_i(r_i)) - \lambda_2\left(\sum_{i=1}^{N} P_i - P_t\right) - \mu_2\left(\sum_{i=1}^{N}\eta_i - 1\right), \quad (16)$$

where λ_2 and μ_2 are Lagrangian multipliers. The KKT conditions are written as

$$\frac{\partial \mathcal{L}_2}{\partial \eta_i} = \frac{aB\log_2\left(1 + \frac{P_i h_i}{d_i^{\alpha}\sigma^2}\right)}{1 + \exp^{a(r_i - b)}} - \mu_2 = 0, \ \forall i \in \bar{N}_R \quad (17)$$

Algorithm 2 UPF Power and BW allocation algorithm for $P2$

1: At each time slot, do the following with $k = 0$, $P_i^{[k]} = P_t/N$ and $\eta_i^{[k]} = B/N$
2: **while** (1) **do**
3: with $P_i^{[k]}$, find μ_2 and $\eta_i^{[k+1]}$ from (17), (18), (8a).
4: with $P_i^{[k]}$, $\eta_i^{[k+1]}$, find λ_2, $P_i^{[k+1]}$ from (19), (20), (8b).
5: **if** $\|\boldsymbol{P}^{[k+1]} - \boldsymbol{P}^{[k]}\| < \epsilon$ and $\|\boldsymbol{\eta}^{[k+1]} - \boldsymbol{\eta}^{[k]}\| < \epsilon$ **then**
6: break;
7: **else**
8: $k = k + 1$
9: **end if**
10: **end while**

$$\frac{\partial \mathcal{L}_2}{\partial \eta_i} = \frac{KB\log_2\left(1 + \frac{P_i h_i}{d_i^{\alpha}\sigma^2}\right)}{\log(1 + Kr_i)(1 + Kr_i)} - \mu_2 = 0, \ \forall i \in \bar{N}_E \quad (18)$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{a\eta_i B h_i/(P_i h_i + d_i^{\alpha}\sigma^2)}{\log(2)(1 + \exp^{a(r_i - b)})} - \lambda_2 = 0, \ \forall i \in \bar{N}_R \quad (19)$$

$$\frac{\partial \mathcal{L}}{\partial P_i} = \frac{K\eta_i B h_i/(P_i h_i + d_i^{\alpha}\sigma^2)}{\log(2)\log(1 + Kr_i)(1 + Kr_i)} - \lambda_2 = 0, \ \forall i \in \bar{N}_E \quad (20)$$

where we have used $\frac{\partial \mathcal{L}_2}{\partial \eta_i} = \frac{\log(U_i(r_i))}{\partial U_i(r_i)}\frac{\partial \mathcal{U}_i(r_i)}{\partial r_i}\frac{\partial r_i}{\partial \eta_i}$ and $\frac{\partial \mathcal{L}_2}{\partial P_i} = \frac{\log(U_i(r_i))}{\partial U_i(r_i)}\frac{\partial \mathcal{U}_i(r_i)}{\partial r_i}\frac{\partial r_i}{\partial P_i}$. A numerical algorithm to find a local optimal solution for problem $P2$ is presented in Algorithm 2. Let $\eta_i'^*$ and $P_i'^*$ for $i \in \overline{N}$ denote a solution of $P2$. In order to find $\eta_i'^*$'s and μ_2, which are $N + 1$ unknown variables, we numerically solve (17), (18) together with (8a), which are N nonlinear equations and one linear equation. This is given at the third step of Algorithm 2. Similarly to find $P_i'^*$'s and λ_2, we numerically solve (19), (20) together with (8b), which is fourth step of Algorithm 2.

Although the optimization problems $P1$ and $P2$ are non-concave problem, the solutions found by Algorithm 1 and Algorithm 2 can be characterized as a local maximizer with respect to *uniform resource allocation*, i.e., $P_i = \frac{P_t}{N}$ and $\eta_i = \frac{B}{N}$. To see this, we need to show that Algorithm 1 and Algorithm 2 at each step k improve the objective function with $P_i^{[k+1]}$ and $\eta_i^{[k+1]}$ compared to $P_i^{[k]}$ and $\eta_i^{[k]}$ starting from $P_i^{[0]} = \frac{P_t}{N}$ and $\eta_i^{[0]} = \frac{B}{N}$. It is important to note that when $\eta_i^{[k+1]}$ is computed, $P_i^{[k]}$ is regarded as a constant at the third step of both Algorithms. Since we have

$$\frac{d^2\mathcal{U}_i(r_i)}{dr_i^2} = \begin{cases} -ae^{-a(r_i - b)} & i \in \bar{N}_R \\ \frac{-\log(1 + r_{\max})}{(\log(1 + r_i))^2(1 + r_i)} & i \in \bar{N}_E, \end{cases} \quad (21)$$

$$\frac{d^2\log(U_i(r_i))}{dr_i^2} = \begin{cases} \frac{-a^2 e^{-a(r_i - b)}}{(1 + e^{-a(r_i - b)})^2} & i \in \bar{N}_R \\ \frac{-1 - \log(1 + r_i)}{(\log(1 + r_i))^2(1 + r_i)^2} & i \in \bar{N}_E, \end{cases} \quad (22)$$

which is negative, $\mathcal{U}_i(r_i)$ and $\log(U_i(r_i))$ are concave in r_i. Furthermore, r_i in (3) is an affine function in η_i. Due to the fact that the composite function of a concave function and an affine function is concave [12], $\mathcal{U}_i(r_i)$ and $\log(U_i(r_i))$ are concave in η_i when P_i is taken as a constant. Thus, $\eta_i^{[k+1]}$ improves the objective function compared to $\eta_i^{[k]}$. On the other hand, since r_i is concave in P_i if $\eta_i^{[k]}$ is fixed as a constant, we can see that $\mathcal{U}_i(r_i)$ and $\log(U_i(r_i))$ are concave in P_i due to the fact that the

composite function of two concave functions is concave [12]. Thus, at the fourth step of both Algorithms, where $\eta_i^{[k+1]}$ is regarded as a constant, we can show that $P_i^{[k+1]}$ newly found improves the objective function compared to $P_i^{[k]}$. It can be also expected that the sequences $P_i^{[k]}$ and $\eta_i^{[k]}$ converge to P_i^* and η_i^*, respectively, since at each k both algorithms seek an improvement of respective objective functions *alternatively* along P_i and η_i with a stopping condition at the fifth line. This algorithmic approach is often called *alternating direction method* in [11].

B. Schedulability

We define schedulability P_S of the proposed algorithms as

$$P_S = \Pr[r_i \geq r_m] \text{ for } i \in \bar{N}_R. \tag{23}$$

It can be said that a user is *schedulable* by the proposed algorithm with given B and P_t, if $P_S \geq \gamma$, e.g., $\gamma = 0.95$. This is acceptable, since RT users are less sensitive to such small losses compared to delay and even some losses can be recovered by applications. In (23), it is possible for RT users to suffer from some losses due to a low channel gain such that r_m is not guaranteed. In order to see how much Algorithm 1 and Algorithm 2 improve P_S compared to uniform resource allocation as an initial solution, (23) can be expressed as

$$\begin{aligned} P_S &= \mathbb{E}_d\left[\Pr\left[\eta_i B \log_2\left(1 + P_i h_i/(d_i^\alpha \sigma^2)\right) \geq r_m\right]\right] \\ &\overset{(x)}{=} \int_0^R \Pr\left[h_i \geq \left(2^{r_m/(\eta_i B)} - 1\right) {d_i}^\alpha \sigma^2/P_i\right] 2d_i/R^2 \, \mathrm{d}d_i \\ &\overset{(y)}{=} \int_0^R \exp\left(-\left(2^{r_m/(\eta_i B)} - 1\right) d^\alpha \sigma^2/P_i\right) 2d_i/R^2 \, \mathrm{d}d_i \\ &\overset{(z)}{=} 2\mathcal{X}_i^{\left(\frac{-2}{\alpha}\right)}/(R^2\alpha)\left[\Gamma\left(2/\alpha, 0\right) - \Gamma\left(2/\alpha, \mathcal{X}_i R^\alpha\right)\right], \end{aligned} \tag{24}$$

where $\mathcal{X}_i = \left(2^{r_m/(\eta_i B)} - 1\right)\sigma^2/P_i$. In (a) the user-to-BS distance having probability density function $f_D(d) = \frac{2d}{R^2}, 0 \leq d \leq R$, has been evaluated. Equality (y) is obtained by using complementary cumulative density function of exponential distribution having unit mean. In addition to get equality (z), we have used $\int x^m e^{-\beta x^n} dx = \frac{-\Gamma(\xi, \beta x^n)}{(n\beta^\xi)}$, i.e., the identity 2.33.10 in [13]. Here, $m = 1$, $n = \alpha$, $\xi = \frac{(m+1)}{n} = \frac{2}{\alpha}$, and $\beta = \mathcal{X}_i$. When uniform resource allocation is used, i.e., initial solution of Algorithm 1 and Algorithm 2 , we have $P_i = P_t/N$, $\eta_i = 1/N$, and $\mathcal{X}_i = \mathcal{X} = N\left(2^{Nr_m/B} - 1\right)\sigma^2/P_t$. Fig. 2 validates the analytical relation (24) among P_S, B, r_m, and N via simulation for uniform resource allocation. Note that (23) for the proposed algorithm is evaluated with simulations in Section IV.

For a good network performance with required QoS, P_S in (24) for RT user has to be greater than threshold γ, i.e.

$$2\,\mathcal{X}_i^{(-2/\alpha)}/(R^2\alpha)\left[\Gamma\left(2/\alpha, 0\right) - \Gamma\left(2/\alpha, \mathcal{X}_i R^\alpha\right)\right] \geq \gamma. \tag{25}$$

Substituting $\mathcal{X}_i = \mathcal{X}$ in (25), we get a single variable equation which can be solved to get limit on the maximum number of RT users that can be served with required schedulability threshold γ while fulfilling their minimum rate requirements r_m through uniform resource allocation. To get a closed form expression of number of servable RT users, we consider high SNR regime in (24) and use the relation:

$$\mathbb{E}_d\left[\exp\left(-\left(2^{Nr_m/B}\right) N d^\alpha \sigma^2/P_t\right)\right] \geq \gamma. \tag{26}$$

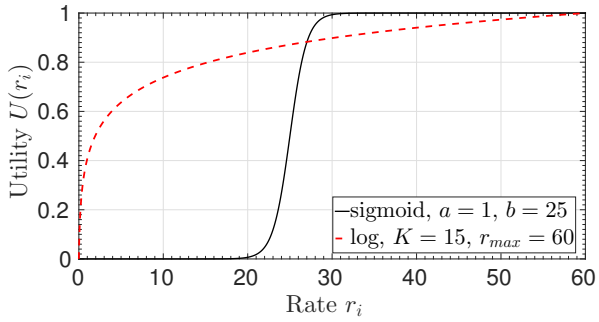

Fig. 1. ET (logarithmic) and RT (sigmoidal) utility functions

Taking log, which is a concave function, on both sides of (26) and using Jensen's inequality $E[f(x)] \leq f(E[x])$ we get: $\left(2^{Nr_m/B}\right) N\sigma^2 R^\alpha/[P_t(\alpha/2+1)\log(1/\gamma)] \leq 1$, which can be represented as $2^{x'}(a'x' + b') = c'$, with solution $x' = W(\ln(2)2^{b'/a'}c'/a')/\ln(2) - b'/a'$; $W(\cdot)$ is the Lambert function. In our case $x' = Nr_m/B$, $c' = 1$, $b' = 0$, and $a' = \sigma^2 R^\alpha B/[P_t\,(\alpha/2+1)\log\,(1/\gamma)\,r_m]$. Thus, we have:

$$N = \lfloor B/[r_m \ln(2)]W(\ln(2)/a')\rfloor. \tag{27}$$

IV. Numerical Results

The ET and RT utility functions used in simulations are represented in Fig. 1, where we set $K = 15$, and $r_{\max} = 60$ for ET, and $a = 1$, $b = 25$ for RT that is suitable for high definition video application [8].

In Figs. 2–3 we set cell radius $R = 250$, a total number of users $N = 14$, and the minimum rate $r_m = 22$. We conducts 200 simulation runs; the locations of users are randomly generated at each run and the number of RT users, N_R, are randomly designated. Moreover, the simulation time of each run is 200 slots. Each time-averaged result over 200 slots is also averaged over 200 runs.

Fig. 2 represents the schedulability P_S in (23); that is, the probability that one RT user experiences rate greater than r_m. For a total of 14 users, we increase RT users from 1 to 14 for $r_m = 22$. The solid lines with symbol $*$ show the schedulability P_s of the proposed Algorithm 1 obtained with simulations, while the dashed lines with symbol $\circ$ depict P_s of proposed Algorithm 2 obtained with simulations too. Further, the dash-dotted lines show analytically obtained P_S in (24), when the uniform allocation is used, and symbol $\times$ represent its corresponding simulation. For $\alpha = 3$, we set $B = 20$ (Hz) and $P_t = 10$ (watt), whereas $B = 25$ (Hz) and $P_t = 15$ (watt) for $\alpha = 3.5$. It can be seen that for $\alpha = 3$, the proposed Algorithm 1 provides 13 RT users with a rate outage up to 5% and performs better than the proposed Algorithm 2 with same available resources. As expected, as pathloss α increases, more BW and power are needed to keep the loss rate low. For $\alpha = 3.5$ even with larger BW and power, the number of RT users with $P_s \geq 0.95$ is reduced to 10 for Algorithm 1. Note that both the algorithms start with initially uniform allocation, i.e., $k = 0$. Although not presented here, when the algorithms start with random initial resource allocation, it yields almost the same results compared to initial uniform allocation.

Fig. 3 shows comparison of the performance of the proposed Algorithm 1 with Algorithm 2. We use $B = 20$, $P_t = 10$, and $\alpha = 3$ for $N = 14$, while N_R increases.

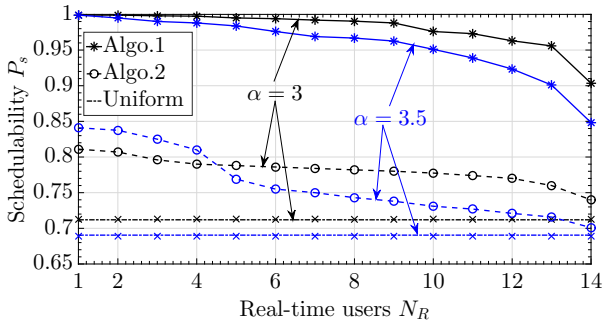

Fig. 2. Schedulability for different number of RT users for $r_m = 22 : 1$) $\alpha = 3$, $P_t = 10$ (watt), $B = 20$ (Hz), and 2) $\alpha = 3.5$, $P_t = 15$ (watt), $B = 25$ (Hz).

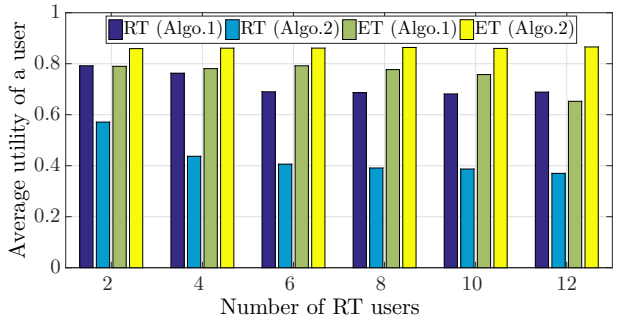

Fig. 3. Comparison of the proposed Algorithm 1 and Algorithm 2 representing the average utility of a user with different number of RT users.

Algorithm 2 assigns more resources to ET users because ET users get higher utility at lower rates, and which severs its main objective of maximizing the sum of log of utility of all users. This results into higher average utility per user for ET in comparison of RT. In proposed Algorithm 1 utility fairness is main objective, and it gives priority to RT users to assign them the minimum required rate. It can be observed from Fig. 3 that to maintain utility fairness and high average utility of RT users, utility of ET users is decreased. Although both the definitions 1 and 2 are utilized for utility proportional fair rate allocation among heterogeneous users [7]–[9], in the wireless network with variable channel conditions, the power and BW resource allocation with UPF definition 1 provides better performance in terms of schedulability and average utility of RT users.The average improvement provided by Algorithm 1 in RT average utility against Algorithm 2 is 70%.

V. Conclusion

We have proposed to jointly optimize power and BW allocation to maximize the UPF among heterogeneous traffic in wireless network having variable channel gain over time. Two optimization problems have been considered corresponding to two UPF definitions. Both the problems being nonconvex, their individual concavity in power and BW has been proved. We have proposed two alternating optimization algorithms for both the problems, and have given insight on their convergence. Instead of maximizing the aggregate network utility, which favours the ET users, the proposed approach provides utility fairness among ET and RT users. Our performance results have shown that the proposed Algo. 1 performs better than the Algo. 2 in terms of schedulability and average utility of RT users.

Appendix

Since the constraints in (7a) and (7b) ((8a) and (8b)) are linear, to see whether the problem $P1$ ($P2$) is a non-concave problem, we need to check nonconcavity of objective function of $P1$ ($P2$). In what follows, it is shown that since $\mathcal{U}(r_i)$ $(\log(U_i(r_i)))$ is quasiconcave in η_i and P_i, the objective function in $P1$ ($P2$), which is sum of quasiconcave functions, is not a concave function, which can complete the proof.

Let us recall that $\mathcal{U}(r_i)$ is concave in r_i from (21), and $\log(U_i(r_i))$ is concave in r_i from (22). Since r_i is a product of affine function $\eta_i B$ and concave function $\log_2\left(1+\frac{P_i h_i}{{d_i}^{\alpha}\sigma^2}\right)$, r_i is a quasiconcave function in η_i and P_i due to the fact that the product of an affine function and a concave function is quasiconcave (Table 5.2 in [14]). Suppose now the composite of concave function f and quasiconcave function g, i.e., $h(x) = f(g(x))$ for $x \in X$. For $x_1, x_2 \in X$, we can write

$$\begin{aligned} h(\theta x_1 + (1-\theta)x_2) &= f(g(\theta x_1 + (1-\theta)x_2)) \\ &\geq f(\min(g(x_1), g(x_2))) = \min(f(g(x_1)), f(g(x_2))) \\ &= \min(h(x_1), h(x_2)), \end{aligned}$$

where quasiconcavity of $g(x)$ has been used. Thus, $h(x)$ is quasiconcave, so is $\mathcal{U}_i(r_i)$ $(\log(U_i(r_i)))$ in η_i and P_i.

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