Energy Efficient Receiver-End Distributed Beamforming Using Orthogonal Transmissions

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***Abstract*—A novel orthogonal transmission based receiver-end distributed beamforming (DBF) scheme, *Rx-DBF* is proposed, for information transfer without receiver feedback. It is proven to achieve near-perfect DBF gain and is channel fading agnostic. Exploiting the correlation in beamformed output, a *sporadic receiver feedback scheme* is also introduced, which prolongs the duration of DBF gain to aid information or power transfer. The proposed schemes are demonstrated to be more energy efficient compared to the closest competitive approaches in the literature.**

***Index Terms*—Distributed beamforming (DBF), fading channel, phase synchronization, orthogonal transmissions**

I. Introduction

Distributed beamforming (DBF) involves collaboration among N independent wireless transmitter nodes for upto N^2 times signal-to-noise ratio (SNR) gain at the receiver as compared to a single transmitter. This requires synchronization among the transmitter signals in time, frequency, and phase, without which, the average SNR gain is limited to N due to random constructive and destructive interference. Phase synchronization in DBF is quite challenging. Unlike classical beamforming, here each transmitter has its own local oscillator (LO) with random phase offset. Also, fading channels between transmitters and receiver introduce dynamic phase offset among received signals. The challenge is aggravated when participating nodes are energy constrained, as frequent signaling among them for synchronization is infeasible. Further, receiver may have limited transmission capability; so reliance on receiver feedback for synchronization needs to be reduced in such wireless charging or data communication applications.

A. Related works and motivation

The existing phase synchronization techniques which can be classified as transmitter-centric and receiver-centric require significant overhead. Many transmitter-centric approaches ([1]–[6]) rely on receiver feedback. [1] needs at least $5N$ iterations to achieve 75% of ideal beamforming gain with each iteration having $N+1$ transmissions and receiver feedback. Continuous coordination among transmitters is required in [2] for inter-node ranging, whereas [3] requires additional hardware. While [5], [6] need N transmissions and feedback, the sequential approach in [4] requires MN slots for convergence and N feedback from receiver with $M > 4$ for

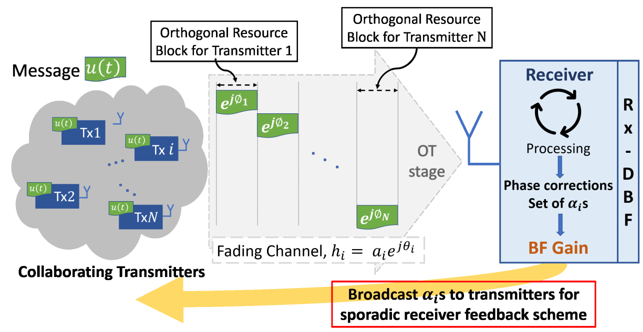

Fig. 1: Distributed beamforming network.

high beamforming gain. Receiver-centric methods are limited in number. [7] requires more than 50 retransmissions for 3 transmitters; increasing exponentially with N. [8] requires N^2 transmissions and has unstable gain in fading environments. Moreover, [7], [8] do not offer appreciable energy gain. The transmitter-centric methods have large overhead, as they correct phase at the transmitters instead of the receiver where synchronization is finally required; this also necessitates receiver feedback. Hence, a receiver-end method would be more suitable. Further, many methods involve all transmitters simultaneously to quickly synchronize, whereas, it would be more energy efficient to correct phase of each signal individually. To this end, this letter proposes a novel scheme for receiver-end DBF phase synchronization based on orthogonal transmissions. The proposed approach isolates the collaborating transmitters' signals using orthogonal transmissions and yet achieves coherent addition of signals at the receiver.

B. Contributions and significance

The key contributions are: (1) An efficient *Rx-DBF* is proposed for phase synchronization using one transmission per transmitter and no receiver feedback. Mathematical analysis and simulations show that it achieves near-perfect and fading agnostic beamforming gain. (2) A *sporadic receiver feedback* based extension is presented for sustained beamforming over extended periods by tracking beamformed output; increased gains are quantified via analysis and simulations. The proposed methods would be impactful in energy efficient collaborative wireless information or power transfer, especially for energy-constrained nodes and when receiver feedback is difficult.

II. Proposed Receiver-End DBF Methods

A. System model

Fig. 1 shows the system model with N collaborating transmitters and a target receiver, each with a single antenna and

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placed at an unknown location. The objective is to transfer data or power to the receiver via DBF, to achieve SNR gain by N^2; for this phase synchronization is critical. There are multiple sources of phase de-synchronization among transmitter signals: $\phi_i \sim U[-\pi, \pi)$ is the initial phase offset of the transmitters' independent LOs. θ_i is the phase shift by channel $h_i = a_i e^{j\theta_i}$ between transmitter i and receiver. Timing and frequency synchronization of the transmitters are done through out-of-band channel. Depending on the stage of the proposed DBF methods, the transmitters transmit in either *orthogonal mode* or *phase synchronous mode*. In orthogonal mode, each transmitter uses mutually exclusive orthogonal resources as shown in Fig. 1. Then the receiver computes phase corrections, and Rx-DBF achieves beamforming without receiver feedback. In the sporadic receiver feedback based extension, the phase corrections are notified to the transmitters. The transmitters then use phase synchronous mode with simultaneous transmissions to achieve direct beamforming at the receiver for extended duration. These methods are detailed in the subsequent subsection. The efficacy of the proposed phase synchronization is analyzed in terms of normalized beamforming gain (G), i.e., the ratio of the realized beamformed power and the ideal beamformed power. Let P_i be the received power from the ith transmitter and P_o be the beamformed power generated by the proposed algorithm. Then, $G = \frac{P_o}{(\sum_{i=1}^{N} \sqrt{P_i})^2}$.

B. Proposed Rx-DBF method

The motivation for Rx-DBF is to use a single correction for a transmitted signal's accumulated phase error at the receiver, instead of individually extracting and correcting phase errors from different sources. The algorithm comprises of two stages (cf. Fig. 2): *orthogonal transmission (OT) stage* when receiver collects transmitters' signals via orthogonal transmissions, and *beamforming (BF) stage* when receiver computes phase corrections and applies them to the corresponding received signals to generate beamformed output. These stages can be repeated cyclically as required. The stages are detailed below:

1) ***OT stage****:* In this stage, the transmitters send the common message signal orthogonally, using TDMA, FDMA, OFDMA, CDMA, or other orthogonal resources. Total N orthogonal transmissions are required, one from each transmitter. Thus, the proposed phase synchronization scheme does not require communication overhead as compared to conventional peer-to-peer communications. Further, the information transfer does not need to be preceded by pilot signals. Transmission matrix is shown in (1), where the rows represent the transmitters and columns represent the orthogonal resources:

$$T_N = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix} \triangleq I_N. \quad (1)$$

These orthogonal signals are distinguishable at the receiver as

$$r_i'(t) = \Re\{h_i p_i e^{j2\pi f_{c_i} t}\} = \Re\{a_i A_i e^{j2\pi f_{c_i} t + j\phi_i + j\theta_i}\} \quad (2)$$

where $p_i = A_i e^{j\phi_i}$ is the complex low-pass equivalent signal and f_{c_i} is the carrier frequency. Here, the received baseband

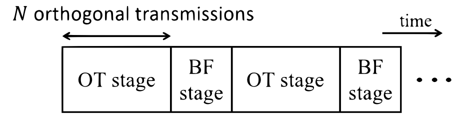


Fig. 2: Rx-DBF: OT stage followed by BF stage.

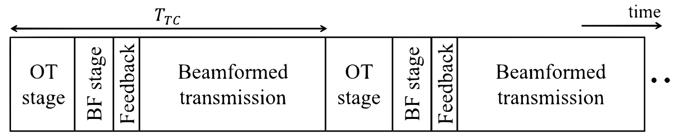


Fig. 3: Prolonged beamformed transmission in sporadic feedback.

signal of ith transmitter is $\tilde{s}_i = e^{j(\phi_i+\theta_i)}$ $\quad 1 \le i \le N$. a_i and A_i have been considered to be unity for simplicity, as they do not have any impact on the phase synchronization process.

2) ***BF stage****:* In this stage, each of the transmitters' signal is pairwise beamformed with an arbitrary reference signal $\tilde{s}_0 = e^{j\phi_0}$ at the receiver to find an optimal required phase shift α_i for the signal from transmitter i, $1 \le i \le N$. Mathematically, the optimal phase shift is given by $\alpha_i = \phi_0 - (\phi_i + \theta_i)$. The challenge in finding optimal value of α_is is non-trivial as the phase error cannot be determined unless measured explicitly. To this end, the following algorithmic method is used:

The ith transmitter's signal is shifted by all possible values from 0 to 2π radians with a selected *step size* of δ radians and added to the reference signal. That α_i is considered optimum which results in the maximum combined signal strength, i.e.,

$$\alpha_i = \underset{\tilde{x}_i}{arg\,max} \left| \tilde{s}_0 + \tilde{s}_i e^{j\tilde{x}_i} \right| = \underset{\tilde{x}_i}{arg\,max}\ C_i(\tilde{x}_i). \quad (3)$$

With these optimal phase shifts, all the received signals from the transmitters are added together at the receiver after applying the corresponding computed optimal phase shifts to get the beamformed signal, as shown in (4).

$$S = \sum_{i=1}^{N} \tilde{s}_i e^{j\alpha_i} = \sum_{i=1}^{N} e^{j(\phi_i+\theta_i+\alpha_i)} \quad (4)$$

C. Sporadic receiver feedback scheme

The Rx-DBF achieves beamforming gain with reduced inter-node interaction to conserve energy. However, applications such as wireless power transfer require continuous DBF for appreciable energy gains. Rx-DBF does not cater to this scenario as during OT stage, there is no gain. In this section, a sporadic receiver feedback scheme is proposed for appreciable energy gains by providing sustained beamformed gains. It works as follows: After the proposed Rx-DBF is executed to obtain the optimum phase shifts, the receiver feeds back the computed optimum phase shifts to the transmitters in one broadcast transmission as shown in Fig. 3. The transmitters then proceed to transmit in phase synchronous mode (instead of orthogonal mode) after pre-applying their corresponding phase corrections. As a result, the phase errors are compensated at the transmitter itself and beamforming is directly achieved at the receiver. This is termed as *beamformed transmission stage* and can be continued for sustained beamforming without repeating OT stage or phase computations. Note that, this scheme assumes the channel coherence time is larger than the duration of the phase alignment process.

Further, when FDMA or OFDMA is employed in the OT stage, channel coherence bandwidth is considered to be larger than the range of frequencies involved. This enables the phase shifts computed in OT stage to be applied in the beamformed transmission stage.

Over time, due to channel dynamics, the earlier calculated phase shifts become non-optimal and beamforming gain reduces; the duration, depends on the mobility environment. When beamforming gain reduces below an acceptable threshold (at time T_{T_C}), OT stage is re-initiated to recalculate optimum phase shifts at the receiver as per the prevailing channel conditions. This is followed by the BF stage and feedback, and beamformed transmission is resumed till the gain remains above the acceptable threshold. Thus, the sporadic receiver feedback based scheme dynamically tracks the beamformed output to maximize the duration of beamformed transmission.

III. Performance Analysis

In this section we analytically capture the achievable beamforming gains with the proposed DBF strategies.

A. Analysis of Rx-DBF performance

Let x_i be the phase error of ith transmitter's signal in alignment with ϕ_0 after Rx-DBF is executed. Then, the final phase of ith transmitter's signal is $\Phi_i = \phi_i+\theta_i+\alpha_i = \phi_0+x_i$.

1) Average gain, G_A: With the phase alignment error, normalized output power P_N of Rx-DBF is expressed as

$$NP_N = \left|\sum_{i=1}^{N} e^{j\Phi_i}\right|^2 = \left|\sum_{i=1}^{N} e^{j(\phi_0+x_i)}\right|^2 = \left|\sum_{i=1}^{N} e^{jx_i}\right|^2. \quad (5)$$

Using Proposition 2 in [9], expected beamformed power is

$$\mathbb{E}[P] = N \times \mathbb{E}[P_N] = N\left[1+(N-1)(\mathbb{E}[\cos x_i])^2\right]. \quad (6)$$

By Taylor series expansion of $\cos x$ and omitting higher order terms, $\mathbb{E}[P]$ is expressed in terms of variance $\mathrm{Var}(x_i)$ when probability density function of x_i is symmetric about zero:

$$\mathbb{E}[P]=N\left[1+(N-1)\left(1+\frac{(\mathrm{Var}(x_i))^2}{4}-\mathrm{Var}(x_i)\right)\right]. \quad (7)$$

In the proposed Rx-DBF algorithm, the distribution of x_i is restricted in $\left[-\frac{\delta}{2}, \frac{\delta}{2}\right]$. Hence, $\mathrm{Var}(x_i)$ is limited to $\frac{\delta^2}{4}$ in the worst case irrespective of the distribution of x_i. Therefore, the lower bound of average beamforming gain is obtained as

$$G_A = \frac{1+(N-1)\left(1+\frac{\delta^4}{64}-\frac{\delta^2}{4}\right)}{N} \quad (8)$$

Remark 1: From (8) it is apparent that the average beamforming gain G_A depends only on step size δ and N, not on the channel phase shift θ_i. Hence G_A is fading agnostic.

2) Worst case gain, G_W: In (5), $|x_i|$ cannot be greater than $\frac{\delta}{2}$ as δ is the step size. Thus, finding worst case beamformed power leads to the following minimization problem:

$$\begin{aligned} &\textbf{(P1)}: \min_{x_i} \left|\sum_{i=1}^{N} e^{jx_i}\right|^2 = \min_{x_i} f(\boldsymbol{x}) \\ &\text{s. t.} \quad \textbf{(C1)}: -\frac{\delta}{2} \le x_i \le \frac{\delta}{2} \quad \forall i \in \{1,\cdots,N\} \end{aligned} \quad (9)$$

where $f(\boldsymbol{x}) = \left(\sum_{i=1}^{N}\cos x_i\right)^2 + \left(\sum_{i=1}^{N}\sin x_i\right)^2$.

Using slack variables s_i and t_i in the constraints,

$$\frac{\delta}{2} - x_i - s_i^2 = 0, \;\; \frac{\delta}{2} + x_i - t_i^2 = 0 \;\; \forall i \in \{1,\cdots,N\}. \quad (10)$$

Forming the corresponding Lagrangian $\mathcal{L}$,

$$\begin{aligned} \mathcal{L} =& \left(\sum_{i=1}^{N}\cos x_i\right)^2 + \left(\sum_{i=1}^{N}\sin x_i\right)^2 \\ &+ \lambda_{11}\left(\frac{\delta}{2}-x_1-s_1^2\right)+\cdots\lambda_{1N}\left(\frac{\delta}{2}-x_N-s_N^2\right) \\ &+ \lambda_{21}\left(\frac{\delta}{2}+x_1-t_1^2\right)+\cdots\lambda_{2N}\left(\frac{\delta}{2}+x_N-t_N^2\right) \end{aligned} \quad (11)$$

where $\lambda_{1i}, \; \lambda_{2i} \ge 0 \;\forall i \in \{1,\cdots,N\}$ are the Lagrange multipliers. Differentiation with respect to x_i, s_i, and t_i give

$$2\sum_{k=1}^{N}\cos x_k \sin x_i - 2\sum_{k=1}^{N}\sin x_k \cos x_i + \lambda_{1i} - \lambda_{2i} = 0, \quad (12)$$

$$-2\lambda_{1i}s_i = 0, \quad -2\lambda_{2i}t_i = 0. \quad (13)$$

From (10) and (13), we have the following possibilities: For $s_i \neq 0$, $\lambda_{1i} = 0$, we have $x_i = \frac{\delta}{2} - s_i^2 < \frac{\delta}{2}$, whereas if $s_i = 0$, $\lambda_{1i} \neq 0$, we have $x_i = \frac{\delta}{2}$. For $t_i \neq 0$, $\lambda_{2i} = 0$, we have $x_i = -\frac{\delta}{2} + t_i^2 > -\frac{\delta}{2}$, whereas if $t_i = 0$, $\lambda_{2i} \neq 0$, we have $x_i = -\frac{\delta}{2}$. Based on this, we can have three cases, Case 1: $x_i = \frac{\delta}{2}$, Case 2: $x_i = -\frac{\delta}{2}$, and, Case 3: $-\frac{\delta}{2} < x_i < \frac{\delta}{2}$ when $\lambda_{1i} = \lambda_{2i} = 0$. Placing Case 3 in (12) and solving, we have

$$\tan x_i = \frac{\sum_{k\neq i}^{N}\sin x_k}{\sum_{k\neq i}^{N}\cos x_k} \quad \forall N \ge 2. \quad (14)$$

However, for $N = 2$, we have $\tan x_1 = \tan x_2$. The only solution to satisfy **(C1)** is $x_1 = x_2$, but it maximizes **(P1)** by aligning all signals. Hence, we consider only Cases 1 and 2, i.e., $|x_i| = \frac{\delta}{2} \;\forall i \in \{1,\cdots,N\}$ for minimizing **(P1)**. For the worst case, the individual sign of each x_i needs to be ascertained. To this end, a secondary optimization problem **(P2)** is defined based on the results of **(P1)**, as

$$\begin{aligned} &\textbf{(P2)}: \min_{n} \left|ne^{j\frac{\delta}{2}} + (N-n)e^{-j\frac{\delta}{2}}\right|^2 \\ &\text{s. t.} \quad \textbf{(C2)}: 0 \le n \le N;\, n \in \mathbb{Z}. \end{aligned} \quad (15)$$

This is a mixed-integer programming problem and is NP-hard. So, linear programming (LP) relaxation technique is used to remove the integer constraint on n and then optimal solution is investigated. Rewriting objective function in **(P2)** as $g(n)$,

$$\begin{aligned} g(n) &= \left|e^{-j\frac{\delta}{2}}\right|^2 \left|ne^{j\delta}+N-n\right|^2 \\ &= (N+n(\cos\delta-1))^2 + n^2\sin^2\delta. \end{aligned} \quad (16)$$

Taking derivative of $g(n)$ and solving, we have $n = \frac{N}{2}$. This value of N satisfies **(C2)** when N is even. For odd values of N, rounding off to the nearest integer values (as per LP relaxation) we get $n = \frac{N+1}{2}$ or $n = \frac{N-1}{2}$. Further, from the second derivative, $g''(n) = (\cos\delta-1)^2 + \sin^2\delta > 0$, the minimizing nature of the solution is confirmed.

Hence, the condition for the occurrence of worst case beamforming gain G_W can be summarized as follows: When

N is even, $x_i = \frac{\delta}{2}$ for $\frac{N}{2}$ number of phase errors $\{x_i\}$ and $x_i = -\frac{\delta}{2}$ for the rest $\frac{N}{2}$ number of phase errors. When N is odd, the worst case beamforming gain can occur under two circumstances; if $x_i = \frac{\delta}{2}$ for $\frac{N-1}{2}$ number of values of $\{x_i\}$ and other $\frac{N+1}{2}$ values are $x_i = -\frac{\delta}{2}$, OR if $\frac{N+1}{2}$ number of values of $\{x_i\}$ are $\frac{\delta}{2}$ and other $\frac{N-1}{2}$ values of $\{x_i\}$ are $-\frac{\delta}{2}$. Thus, the expressions for G_W are derived as follows:

1) N is even

$$\begin{aligned} G_{W_e} &= \frac{\left|e^{j\frac{\delta}{2}} + e^{-j\frac{\delta}{2}} + \cdots + e^{j\frac{\delta}{2}} + e^{-j\frac{\delta}{2}}\right|^2}{N^2} \\ &= \frac{\left|\frac{N}{2} \times 2\cos\frac{\delta}{2}\right|^2}{N^2} = \cos^2\frac{\delta}{2}. \end{aligned} \tag{17}$$

2) N is odd

$$\begin{aligned} G_{W_o} &= \frac{\left|e^{j\frac{\delta}{2}} + e^{-j\frac{\delta}{2}} + \cdots + e^{j\frac{\delta}{2}} + e^{-j\frac{\delta}{2}} + \cdots + e^{\pm j\frac{\delta}{2}}\right|^2}{N^2} \\ &= \frac{\left|(N-1)\cos\frac{\delta}{2} + e^{\pm j\frac{\delta}{2}}\right|^2}{N^2} = \cos^2\frac{\delta}{2} + \frac{\sin^2\frac{\delta}{2}}{N^2}. \end{aligned} \tag{18}$$

Remark 2: Thus, it is notable that the worst case beamforming gain G_W depends only on step size δ; G_W is independent of channel phase shift θ_i, hence it is channel fading agnostic.

B. Performance with sporadic receiver feedback

Here, the total gain in the beamformed transmission stage is evaluated. To estimate the duration of beamformed transmission stage, a model is proposed for the combined received signal from multiple transmitters in DBF by comparing it with the combined reception of multipath components (MPC) from a single transmitter over a narrowband fading channel.

The combined signal at the receiver in DBF is given by

$$r_{DBF}(t) = \sum_{i=1}^{N} a_i u(t) e^{j(2\pi f_c t + \psi_i(t))}. \tag{19}$$

Here a_i is the amplitude, f_c is the carrier frequency, $u(t)$ is the complex low-pass equivalent of the transmitted signal, and ψ_i is the phase which varies due to fading.

On the other hand, combined received signal in a multipath fading channel can be expressed as

$$r_{MPC}(t) = \sum_{m=1}^{M} b_m u(t-\tau_m) e^{j(2\pi f_c (t-\tau_m) + \psi_{D_m}(t))} \tag{20}$$

where there are M resolvable multipath components with amplitude b_m, delay τ_m and $\psi_{D_m}(t) = 2\pi f_{D_m} t$ is the Doppler phase shift for Doppler frequency f_{D_m}.

$r_{MPC}(t)$ of (20) and $r_{DBF}(t)$ of (19) can be shown to have similar structure under the following justifiable assumptions: (a) The different MPC reach the receiver almost simultaneously due to small delay spread ($\tau_m \approx 0$) of the narrowband fading channel. Similarly, the signals from the collaborating transmitters in DBF arrive at the same time at the receiver due to timing synchronization. (b) Non-dominance of LOS signal in MPC. This is a reasonable assumption in DBF where the transmitters are large distance away from the receiver such that none of the transmitter signals are dominant over the others.

By these assumptions, the expression for $r_{MPC}(t)$ becomes

$$r_{MPC}(t) = \sum_{m=1}^{M} b_m u(t) e^{j(2\pi f_c t + 2\pi f_{D_m} t)}. \tag{21}$$

The similarity of (21) and (19) is evident. Thus, the effect of number of MPC from one transmitter to a receiver is analogous to receiving from multiple independent transmitters in DBF.

Accordingly, the well-known results of multipath channels are applied to the current DBF problem. In multipath fading, envelope of r_{MPC} is Rayleigh [10]. By comparison, the envelope of the beamformed output in DBF is also Rayleigh distributed. Let us apply some additional analogies. (1) Doppler phase shift of a MPC is considered independent of its amplitude. Likewise in DBF, amplitude and phase are independent. (2) Phase shifts between two MPCs are independent. Likewise, phase shifts on two different transmitter signals in DBF are considered independent. Then, using Clarke's model [11], auto-correlation function of the beamformed output is given by: $R_{BF}(\tau) = P_0 J_0(2\pi f_D \tau)$ where $P_0 = \frac{N\,\mathbb{E}[a_i^2]}{2}$. A simple Markov chain model for Rayleigh channel [12] is applied to the beamformed output in DBF:

$$r_{DBF}(t) = J_0(2\pi f_D t) r_{DBF}(t_0) + \sqrt{1 - J_0(2\pi f_D t)^2}\,\varepsilon \tag{22}$$

where $\varepsilon \sim \mathcal{CN}(0,1)$, $t > t_0$, $r_{DBF}(t_0) = N$ (ideal), because at the start of beamformed transmission stage, transmitter phase shifts are optimal.

Let A_{th} be a given threshold of beamformed amplitude that is required for meeting the application needs. The beamformed transmission stage ends when $|r_{DBF}(t)| = A_{th}$, i.e., $A_{th} = \left|N J_0(2\pi f_D t) + \sqrt{1 - J_0(2\pi f_D t)^2}\varepsilon\right|$. To derive the mean threshold crossing time interval T_{TC}, consider $\varepsilon = \varepsilon_X + j\varepsilon_Y$ where $\varepsilon_X, \varepsilon_Y \sim \mathcal{N}(0, 0.5)$ so that

$$\begin{aligned} A_{th} &= \left|N J_0(2\pi f_D t) + \sqrt{1 - J_0(2\pi f_D t)^2}(\varepsilon_X + j\varepsilon_Y)\right| \\ \text{or, } A_{th}^2 &= (N J_0(2\pi f_D t) + \sqrt{1 - J_0(2\pi f_D t)^2}\varepsilon_X)^2 \\ &\quad + (1 - J_0(2\pi f_D t)^2)\varepsilon_Y^2. \end{aligned} \tag{23}$$

Applying the expectation operator on both sides of (23),

$$\begin{aligned} A_{th}^2 = &(1 - J_0(2\pi f_D T_{TC})^2)(\mathbb{E}[\varepsilon_X^2] + \mathbb{E}[\varepsilon_Y^2]) + N^2 J_0(2\pi f_D T_{TC})^2 \\ &+ 2N J_0(2\pi f_D T_{TC})\sqrt{1 - J_0(2\pi f_D T_{TC})^2}\,\mathbb{E}[\varepsilon_X]. \end{aligned} \tag{24}$$

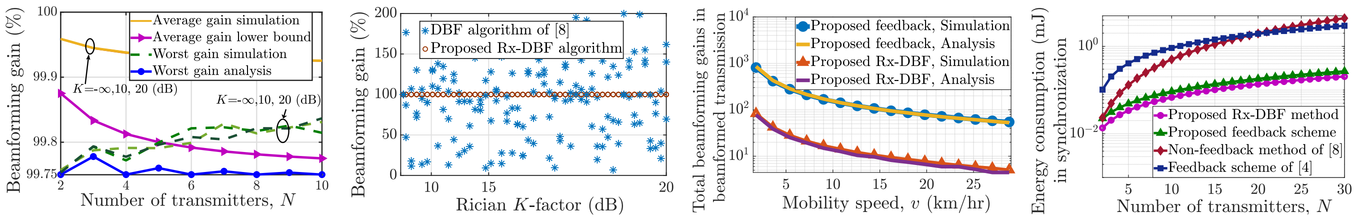
Using power series approximation of Bessel function of first kind and zeroth order, and neglecting the higher order terms

$$J_0(2\pi f_D T_{TC}) = 1 - \frac{4\pi^2 f_D^2 T_{TC}^2}{4} = \sqrt{\frac{A_{th}^2 - 1}{N^2 - 1}}. \tag{25}$$

Solving for T_{TC} and using $f_D = v/\lambda$

$$T_{TC} = \left(1 - \sqrt{\frac{A_{th}^2 - 1}{N^2 - 1}}\right)^{\frac{1}{2}} \frac{\lambda}{\pi} \times \frac{1}{v}. \tag{26}$$

Having derived the duration of beamformed transmission stage, total gain over the interval of mean threshold crossing time is evaluated for the feedback scheme versus the Rx-DBF algorithm. Taking slot duration $= \frac{k}{f_c}$, where k is some integer, the number of slots in mean threshold crossing time is: $N_{TC} = \frac{T_{TC} f_c}{k}$. Therefore, total gain over T_{TC} in the feedback scheme



(a) Average and worst case beamforming gain with the proposed Rx-DBF **(b)** Effect of fast fading on DBF gain, with $N = 2$ **(c)** Increased total gains with sporadic feedback, for $N = 10$, $A_{th} = 0.7N$ **(d)** Energy consumption in synchronization of the proposed methods

Fig. 4: Performance of the proposed receiver-end DBF methods.

is the average gain over N_{TC} slots, given by $G_{N_{TC}} = \frac{\mathbb{E}[P]+A_{th}^2}{2N^2}$ multiplied by the number of slots, i.e.,

$$Gain(\text{with feedback}) = G_{N_{TC}} \cdot N_{TC}. \tag{27}$$

In comparison, without feedback, using TDMA based OT, beamformed output is obtained in every $(N+1)^{th}$ slot due to repeated OT stage. Hence, the total gain in this case is

$$Gain(\text{without feedback}) = \frac{G_{N_{TC}}}{(N+1)} \cdot N_{TC}. \tag{28}$$

Remark 3: Hence, using sporadic feedback, beamforming gain is increased by N times with respect to without feedback.

IV. Results and Discussion

We now present the performance results of the proposed DBF schemes and compare with the closest competitive approaches in literature. Common system parameters considered in numerical and simulation studies are $f_c = 915$ MHz, $\delta = 0.1$ rad and slot size $T_S = 100$ μs (packet size 20 Bytes and data rate 1.6 Mbps). Other parameters are stated along with the discussion of individual performance results.

Fig. 4a shows the average as well as worst case beamforming gains with the proposed Rx-DBF. It is observed that near-perfect average beamforming gain is obtained irrespective of the number of transmitters N. It can also be seen that the plot of analytical lower bound of the average gain in (8) has a maximum deviation of $< 0.2\%$ with respect to simulation results. The fading agnostic nature of the average and worst case beamforming gains, analytically captured respectively in (8), (17), and (18), are confirmed by the simulation results which do not show variation with varying Rician-K factor which signifies severity of fading. The increasing trend of worst case in the simulation is due to decreasing probability of hitting the worst case when N increases. Further, Fig. 4b shows that Rx-DBF is also robust under fast fading channel, whereas the nearest competitive strategy in [8] shows large variation in beamforming gain. We have also observed that δ up to ~ 0.2 radians achieves near-perfect beamforming gain.

Fig. 4c shows a significant increase in beamforming gain by using sporadic feedback. A good match of the analytical results with the simulations verify correctness of the analysis.

To compare energy efficiency of the proposed Rx-DBF and sporadic receiver feedback based enhancement with the most competitive works without and with feedback respectively, the total communication energy spent in DBF is plotted in Fig. 4d. Here, the transmission and reception energy requirements of CC1310 wireless MCU are used. As observed, *energy consumption of the proposed methods are much lower than the competitive non-feedback [8] and feedback [4] schemes*.

V. Conclusion

In this work, an energy efficient Rx-DBF technique for phase synchronization has been presented that avoids any interaction among the participating nodes to conserve energy. Mathematical analysis and simulations have shown that it achieves near-perfect beamforming gain and is channel fading agnostic. A sporadic receiver feedback scheme has also been proposed that exploits temporal correlation of beamformed output to achieve extended gain without repeated orthogonal transmissions and processing at receiver. The beamforming gains with feedback have been analytically quantified. The proposed methods have been shown to be more energy efficient compared to the closest competitive approaches.

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