On ARQ Based Switched Antenna Diversity in Markov Channels

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Abstract

Since switch and stay combining (SSC) scheme performs at the block level, it can be combined with an automatic repeat request (ARQ) scheme. In this article, performance of an ARQ based SSC scheme in Markov channel is analyzed. It demonstrates the performance benefit with SSC-ARQ and the conditions set on the channel state for the scheme to be feasible.

I. INTRODUCTION

The simple and popular dual-branch switch-and-stay combining (SSC) employs a single receiver but with two antennas [1], [2], [3, and references therein]. Here, the receiver periodically monitors the signal quality (signal-to-noise ratio − SNR) at the current antenna during the 'guard periods'. It continues to receive data blocks at the current antenna till the SNR is above a threshold level. If the SNR falls below the threshold, the receiver switches the antenna branch to continue to receive the next block of data.

The performance of antenna diversity schemes is practically always considered in terms of bit error rate (BER), though this does not provide sufficient information on the packet error rate (PER) − which is a more useful measure at the upper layers. However, since SSC works at the block level, SSC in conjunction with an ARQ scheme is built naturally if correctness of a received block is utilized as the channel state information (CSI) [4], [5]. While both [4] and [5] are simulation studies, in this letter we provide a performance analysis of the SSC-ARQ scheme in a Markov (Gilbert) channel [6]. Note that, while a Markov modelling of the SSC scheme is presented also in [3], the modelling, analysis, results, and the insights obtained are completely different here.

II. ANALYSIS

Consider the receiver is equipped with two antennas R_A and R_B and the transmitter has a single-antenna T_A. The wireless channels between $T_A - R_A$ and $T_A - R_B$ are assumed to be non-identical and independent two-state Markov channels. This channel model is comprised of a 'good' and a 'bad' state, with respective PER values 0 and 1. This is the commonly known Gilbert channel model [6] and extensively used in modelling correlated errors in fading channels [7], [8]. Transition probabilities of the two channels are given by:

$$T_{R_A} = \left(\begin{array}{cc} p_1 & p_3 \\ p_4 & p_2 \end{array} \right) \quad \text{and} \quad T_{R_B} = \left(\begin{array}{cc} q_1 & q_3 \\ q_4 & q_2 \end{array} \right) \tag{1}$$

where $p(q)_3 = 1 - p(q)_1$ and $p(q)_4 = 1 - p(q)_2$.

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PER values over the respective channels are given by:

$$\text{PER}_A = \frac{1 - p_1}{2 - p_1 - p_2} \quad \text{and} \quad \text{PER}_B = \frac{1 - q_1}{2 - q_1 - q_2} \tag{2}$$

Each transmitted block is assumed equipped with a cyclic redundancy check (CRC) for error detection. The SSC-ARQ scheme is then simply stated as: reception at the current antenna continues if a block is received correctly. Otherwise, a negative acknowledgment (NACK) is sent and the antenna is switched to receive the next block. We further assume an infinite transmission buffer, negligible round-trip delay, and error-free return channel providing the acknowledgment.

The system described above is modeled by an 8-state Markov chain as in Fig. 1. The association of the receiver to a particular antenna is indicated by the '*'. Denote the system state at time-step n is $X(n)$. The one-step transition probability P_{ij} is defined as: $P_{ij} = \text{Prob}\{X(n + 1) = j | X(n) = i\}$. We have,

$$P = \begin{bmatrix} p_1 q_1 & p_1 q_3 & p_3 q_1 & p_3 q_3 & 0 & 0 & 0 & 0 \\ p_1 q_4 & p_1 q_2 & p_3 q_4 & p_3 q_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_4 q_1 & p_2 q_1 & p_4 q_3 & p_2 q_3 \\ 0 & 0 & 0 & 0 & p_4 q_4 & p_2 q_4 & p_4 q_2 & p_2 q_2 \\ 0 & 0 & 0 & 0 & p_1 q_1 & p_3 q_1 & p_1 q_3 & p_3 q_3 \\ 0 & 0 & 0 & 0 & p_4 q_1 & p_2 q_1 & p_4 q_3 & p_2 q_3 \\ p_1 q_4 & p_1 q_2 & p_3 q_4 & p_3 q_2 & 0 & 0 & 0 & 0 \\ p_4 q_4 & p_4 q_2 & p_2 q_4 & p_2 q_2 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

The steady state probabilities is obtained by solving the equation $\pi = \pi P$ with the constraining equation $\sum_{l=1}^{8} \pi_l = 1$.

Since only states 1, 2, 5, and 6 of the Markov chain correspond to correct reception of a packet, throughput of the SSC-ARQ combined scheme is given by

$$\eta_{SSC-ARQ} = \pi_1 + \pi_2 + \pi_5 + \pi_6 \tag{4}$$

The solution of (4) in terms of P_{ij} is a very long one, which however gets particularly simplified if we assume the channels are symmetrical (i.e., $p_1 = q_1$ and $p_2 = q_2$), and is given by:

$$\eta_{SSC-ARQ-sym} = \frac{(1 - p_2)^2 + (1 - p_1)(1 - p_2)(p_1 + p_2)}{(2 - p_1 - p_2)^2} \tag{5}$$

To the knowledge of the authors, the analysis and the achieved results in (5) are completely new results.

The throughput of a conventional ARQ system with only one receive antenna is simply given by:

$$\eta_{ARQ} = (1 - \text{PER}) = \frac{1 - p_2}{2 - p_1 - p_2} \tag{6}$$

The throughput gain achieved with SSC-ARQ is:

$$\text{Gain} = \eta_{SSC-ARQ} - \eta_{ARQ} \tag{7}$$

III. Numerical Examples

The gain in SSC-ARQ under symmetric channel assumption is shown in Fig. 2. In the best case, the use of SSC provides a gain of 0.25 (or 50%) over the conventional ARQ scheme.

Notice in Fig. 2 that, SSC-ARQ does not always increase the throughput performance. In fact, the condition that for the 'gain' to be positive and the SSC scheme to be viable is, $2 \geq p_1 + p_2 \geq 1$, i.e., when the channel is slowly varying. Actually the gain peaks at the limit of p_1, $p_2 = 1$, or when the channel is 'quasi-static'. Further, Fig. 3 provides a specific example of how the gain varies with PER for a given value of $p_1 (= 0.8)$. Notice that at low of PER, gain with SSC is insignificant as the need for switching antenna is very low when errors are infrequent. Again at high values of PER, the bad states of the channels are encountered very often even with switching, and the throughput gain becomes again smaller. However, the gain peaks at the intermediate values of the PER.

IV. Conclusion

In this letter, we have provided performance analysis of a receiver switch and stay combining policy in conjunction with an ARQ scheme in Markov channels. The major insight gained from this study is that, the benefits of SSC is better perceived when the channel is slowly varying and with moderate PER values. While these inferences are somewhat intuitive, they are not apparent from the earlier studies on SSC that use BER as the performance measure.

Acknowledgment

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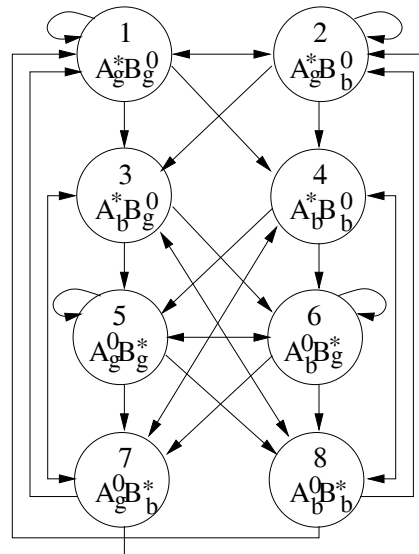
Fig. 1. Markov chain model of the dual antenna SSC-ARQ scheme. A and B denote the respective receive antennas, the subscript (g or b) denote the channel states ('good' or 'bad'), and the superscript ('*' or '0') denote whether the receiver is attached or not.
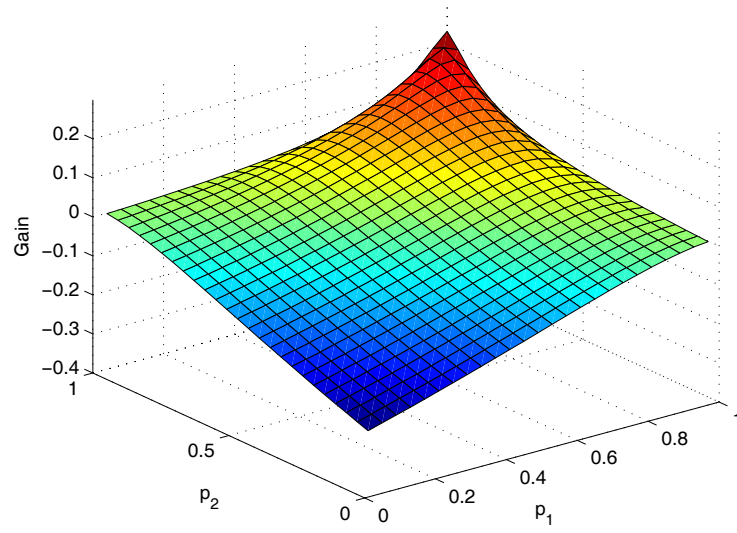
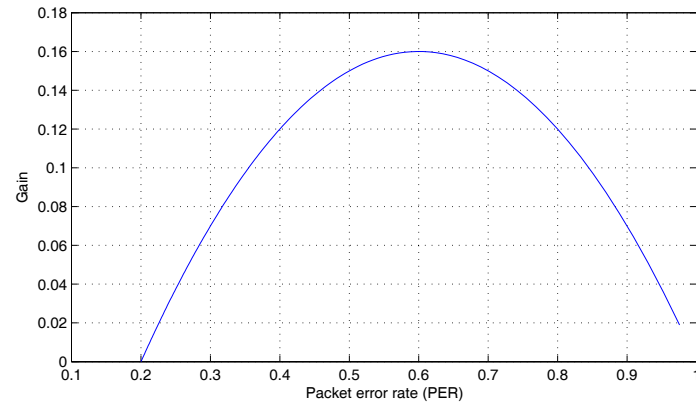Fig. 2. Throughput gain in SSC-ARQ over a conventional ARQ.

Fig. 3. Throughput gain as a function of PER. $p_1 = 0.8$ is chosen, and the peak gain (nearly 0.16) occurs at $p_2 = 0.88$.