On the Characterization of Aloha in Underwater Wireless Networks

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Abstract

Standard analyses of wireless random access protocols that are available in the literature assume negligible propagation delay between any two nodes. This assumption holds good in reasonably short-range terrestrial RF (radio frequency) wireless networks. On the contrary, in wireless communications involving acoustic wave propagation, as in underwater wireless networks, even short distance propagation has appreciably large propagation delay. This observation has led to several recent simulation and experimental studies on underwater Aloha and slotted-Aloha (S-Aloha) protocols and also a few new proposals on random access protocols for underwater wireless ad hoc networks (UWN). To study the efficiency of more advanced multi-access communication protocols for UWN, it is important to benchmark their performances with respect to the two basic random access protocols, Aloha and S-Aloha. This paper provides an analytic framework to capture the performance of Aloha and S-Aloha protocols in an underwater environment with high and random internodal signal propagation delay. The performance of underwater Aloha and S-Aloha are contrasted with those in short-range terrestrial RF wireless networks. The analysis shows that random internodal propagation delay has no effect on the underwater Aloha performance. It also sheds light on the throughput degradation of underwater S-Aloha with a slotting concept that achieves RF S-Aloha equivalent one-slot vulnerability. Additionally, a modified slotting concept is introduced where the slot size is judiciously reduced such that even by allowing some collisions the overall system throughput can be increased. Our calculations show that, with the modified slotting

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approach up to 17% throughput performance gain can be achieved over the naive (RF S-Aloha equivalent) slotting approach in UWN. Our analytic results are supported by discrete event simulations.

1. Introduction

Short-range underwater wireless ad hoc networks (UWN) are aimed at remotely monitoring various aquatic activities, such as marine biological and zoological lives, geological changes, and underwater human activities. There are some similarities in UWN and terrestrial radio frequency (RF) wireless sensor networks, such as, limited channel bandwidth, high bit error rate caused by the wireless channel, and limited battery power of sensor nodes. Therefore, both type of networks have common performance measures, such as, throughput, delay, and battery life. Yet, UWN and terrestrial wireless networks differ in many aspects; propagation delay is the most sensitive parameter of them all. RF networks universally use electromagnetic frequency (EM) waves at various frequency bands. However, due to high attenuation, underwater wireless (UW) communication systems cannot use EM waves. Instead, UW systems use acoustic waves. The atmospheric propagation speed of RF carrier is close to 3×10^8 m/s, that is, speed of light in free space. On the other hand, propagation speed of acoustic waves in normal water is about 1.5×10^3 m/s. Thus, the propagation delay in UW networks is several orders of magnitude higher than that in RF networks. Another important issue is that, the carrier frequency of UW acoustic signals are typically in the range of $1 - 100$ kHz [1], while that of the RF carrier is typically in the range of $0.5 - 60$ GHz. Therefore, the bandwidth of UW networks is also a few orders of magnitude lower than that of the terrestrial wireless networks. This clearly means that the protocols designed for RF networks are unlikely to be directly applicable in UWN [2, 3, 4], necessitating that the network protocols be re-looked for UWN.

Similar to the terrestrial wireless networks, MAC (medium access control) schemes play a very important role in short-range UW wireless networks where the acoustic channel is used as a shared

medium by many nodes. It is well-known that, in a large and random deployment setting with distributed control and bursty data, contention-free access protocols, such as TDMA (time-division multiple access), FDMA (frequency-division multiple access), are not efficient. Moreover, energy-constrained sensor nodes could save energy by physical event-driven communications, which can be a random phenomenon. Therefore, for internodal communications random (contention-based) access protocols are more appropriate.

Many basic random access protocols have been developed for conventional multiple access environments. The basic contention schemes are Aloha and slotted-Aloha (S-Aloha). For longer bursts of data, CSMA and its two important variants – CSMA with collision detection (CSMA/CD) and CSMA with collision avoidance (CSMA/CA) – are used, which provide a combination of random access and reservation. Naturally, these schemes would also be considered for UW multiple access systems. Several recent works on UW multiaccess schemes underline that slight difference of internodal distance has appreciable propagation delay difference, which in turn affects the performance of a random access protocol, and these findings are logical. A few variants of UW network random access protocols have been proposed to mitigate the effects of long propagation delay. We provide here a brief survey of prior works that are pertinent to our current study.

1.1. Prior work

There have been some recent works on UWN multiaccess networks (e.g., [5, 6, 7, 8, 9, 10, 11]). Based on simulation studies of UWN it was suggested in [6] that the maximum performance of S-Aloha is the same as that of Aloha. The effects of internodal propagation delay on *many-to-one* Aloha and S-Aloha throughput performance was studied via simulations in [7]. The Aloha performance was shown to be unaffected by spatial uncertainty. With a *slot size equal to a (fixed) frame transmission time*, their simulation results on S-Aloha showed the throughput degrades to that of Aloha at any propagation delay. Further, to enhance the S-Aloha performance, the authors proposed to increase the slot size by some fractional amount. An analytic study of the many-to-one protocols proposed in [7] was performed in [9]. In [12], two Aloha based variants namely, Aloha with collision avoidance and Aloha with advance notification were proposed, where, a node upon overhearing the neighboring nodes' communication, takes appropriate backoff measure so as to

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minimize the collision probability. It was qualitatively observed in the paper that simple Aloha as an UW random access protocol could be inefficient, but it did not provide any guideline as to how the basic Aloha and S-Aloha protocols would perform under different operating parameters.

To counter the effect of UW propagation delay, RTS/CTS (request-to-send/clear-to-send) based reservation protocol was proposed in [8], where based on the propagation delay of RTS frame and the data length information in it, the receiving node decides a receive window for a collision-free data frame reception. In another work [10], communication between a master (gateway) node and the slave (non-gateway) nodes was considered, where separate channels for control (reservation) and data were suggested in RTS/CTS handshake based reservation protocol. The RTS frames from the non-gateway nodes are sent using the Aloha protocol, and until a desired CTS frame is received at a non-gateway node, it does not transmit its data frame. Note that, such schemes are efficient with relatively longer frames and infrequent transmissions. This process also ensures collision-free data transmission in a single-cell scenario. However, when smaller frames comparable to the size of RTS-CTS frames are transmitted frequently, such explicit reservation mechanisms are clearly not efficient. This is also reflected in the provisions of direct (without RTS/CTS mechanism) data transmission in the IEEE 802.11 standards. Further, the performance of such a scheme may deteriorate in a multi-cell scenario, where a gateway node may be reachable from the nodes outside its cell boundary.

Thus, while reservation based multiaccess protocols, such as CSMA/CA with RTS/CTS, may offer a higher throughput, basic Aloha protocols would be of interest in situations where the return channel for reservation is unavailable or infeasible to use. In other words, basic Aloha protocols are expected to be used in UW communications for short frame transmissions or as a reservation protocol for supporting longer sessions (as in [10], similar to the contention-based channel access in wireless LANs and for paging in the GSM cellular systems).

1.2. Contribution

In this paper, we provide a detailed theoretical basis for the performance evaluation of two basic random access protocols, namely Aloha and S-Aloha for one-to-one communications in underwater environment. Our specific contributions are as follows:

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(a) We derive generalized throughput performance expressions for Aloha with a random intern-odal delay setting, and with fixed as well as variable frame size, which can be used in UWN as well as RF networks. Our analysis and simulations show that pure Aloha performance is indeed independent of signal propagation speed. Note that, while the Aloha-uw performance with a fixed internodal propagation delay is rather apparent, the outcome in a random delay is not so obvious.

(b) When propagation delay is non-negligible, we suggest that the slot size in a randomly de-ployed network be dictated by the maximum propagation delay within a nodal coverage range, which will achieve an equivalence of one slot vulnerability as in traditional S-Aloha-rf. This also imply that, the condition for a higher throughput performance of S-Aloha-uw with respect to Aloha is governed by the nodal coverage range.

(c) To improve the S-Aloha-uw performance we further propose a modified slotting concept, where for a given communication range, slot size can be appropriately chosen as a function of the frame size. Via a closed form analysis supported by simulations we demonstrate that, an optimal choice of slot size can lead up to 17% throughput performance gain with respect to the naive slotting decision. Note that, although the concept of modified S-Aloha presented in the paper is intuitive, the exact analytic proof of throughput performance gain is rather involved.

The objectives in this paper match closely with that of [6], [7], and [9]. However, in contrast with these studies, we provide analyses of *one-to-one* Aloha-uw performance under a random node deployment in an ad hoc network setting, and S-Aloha-uw for any value of internodal propagation delay. Our analysis approach is different from that provided in [9] for many-to-one Aloha proto-cols. The slotting approach in S-Aloha-uw proposed in this paper is different in that, instead of one frame transmission time T_t as the slot size, we propose to have a slot size which is the sum of T_t and the maximum internodal propagation delay $T_p^{\max}$, where $T_p^{\max}$ can be of any value such that $T_p^{\max} <$ or, $=$ or, $> T_t$. To increase the throughput efficiency of S-Aloha-uw, we propose and analyze an optimum slot size reduction factor k. A preliminary version of the work was presented in [13].

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1.3. Paper organization

The remainder of the paper is organized as follows. General assumptions, definitions, and a list of major notations used throughout the paper are provided in Section 2. The throughput performance analysis of Aloha in UW environment is presented in Section 3. Section 4 contains S-Aloha-uw slotting concept that achieves S-Aloha-rf equivalent vulnerability duration, and the throughput analysis. Our proposed modified S-Aloha-uw is presented and analyzed in Section 5. Numerical and simulation results and remarks are provided in Section 6. Section 7 concludes the paper.

2. Assumptions, Definitions, and Notations

The following assumptions and definitions are used in our subsequent discussion.

1. The network consists of homogeneous nodes, with all nodes having equal communication range R. That is, irrespective of the underwater nodes' temporal and spatial locations, nodal coverage range and signal propagation speed are considered fixed.

2. Nodes in the network are uniformly randomly distributed. Besides ensuring that the internodal propagation delay is a random number, uniform random distribution of node locations simplifies the computation of collision probability in modified S-Aloha-uw.

3. Internodal communications are event-driven, which is considered random. This random traffic arrival process, including the backlog retries, is approximated as Poisson distributed with a rate independent of the state of the network. Poisson (memoryless) arrival process with state-independent rate helps simplify the performance evaluation of Aloha protocols.

4. A node outside the communication range is unreachable. Physical channel related frame errors are discounted. A frame can be corrupted and lost due to MAC level collisions only.

5. Temporal variability of internodal propagation delay due to underwater current is not accounted.

6. Throughput performance is measured in terms of *normalized system throughput*, defined as the average number of successful frames in the network per average frame transmission time.

Note that, location and time dependent variability of nodal coverage range and signal propagation speed could be more practical considerations. However, there has not been any suitable model available to characterize them. Also, some other assumptions, namely, 2, 3, and 4 can be relaxed, but the primary claims with these relaxations will remain unchanged, although the analysis will be more complicated.

Major notations used in the paper are listed in Table 1.

3. Aloha in UWN

In this section, we analyze the pure Aloha protocol performance in UWN considering fixed as well as exponentially distributed frame size.

3.1. Aloha-uw with fixed frame size

In Fig. 1, the collision vulnerability windows in short-range Aloha-rf and Aloha-uw multiaccess schemes, respectively, are shown. Note from Fig. 1(a) that, in short-range RF communica-
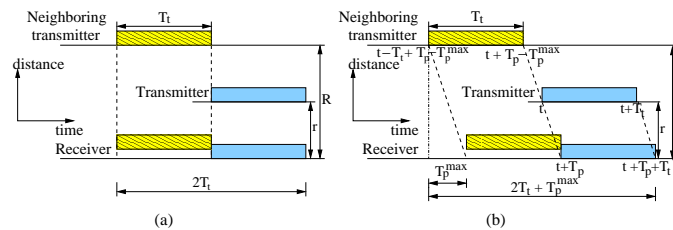

Figure 1: Collision vulnerability window in Aloha protocols. (a) Terrestrial short-range RF wireless network; (b) UW networks.

tions, such as in mobile ad hoc networks, RF wireless sensor networks, and urban cellular wireless networks, where internodal propagation delay is negligible, irrespective of the nodal coverage range, the vulnerability window is $2T_t$. In such RF wireless networks, since the internodal propagation delay of the RF signal is insignificant compared to T_t, the collision probability of a frame is simply the probability of a frame arrival in the window of size $2T_t$. Accordingly, with Poisson distributed traffic arrival process in the network at a total rate λ per unit time, the normalized system

Table 1: Summary of notations

R	Nodal communication (transmit/receive) range
v	Acoustic signal propagation speed
c	RF signal propagation speed
λ	Frame arrival rate in the system per unit time
η	Normalized system (network) throughput
F	Frame length
R_c	Channel rate
T_t	Frame transmission time; $T_t = \frac{F}{R_c}$
T_p	Signal propagation delay, a function of transmitter-receiver distance r; $T_p = \frac{r}{v}$; $r \leq R$
T_p^{max}	Maximum signal propagation delay; $T_p^{\mathrm{max}} = \frac{R}{v}$
T_s^{rf}	Slot size in S-Aloha-rf; $T_s^{\mathrm{rf}} = T_t$
$T_s^{\mathrm{uw}}, T_{s1}$	Slot size in S-Aloha-uw, or slot size in modified S-Aloha-uw with $k = 1$; $T_s^{\mathrm{uw}} = T_t + T_p^{\mathrm{max}} = T_{s1}$
T_{sk}	Slot size in mS-Aloha-uw (modified S-Aloha-uw) with $0 \leq k \leq 1$; $T_{sk} = T_t + kT_p^{\mathrm{max}}$
k	Slot size reduction factor in mS-Aloha-uw; $0 \leq k \leq 1$
r_p	Distance of the receiver from a neighboring transmitter, that has a frame in *previous* slot
r_n	Distance of the receiver from a neighboring transmitter, that has a frame in *next* slot
n_p	Number of frames scheduled in *previous* slot
n_n	Number of frames scheduled in *next* slot

throughput in Aloha-rf scheme with fixed frame size is:

$$\eta_{\mathrm{Aloha\text{-}rf}}^{(\mathrm{fixed})} = \lambda T_t \cdot \Pr[\text{no collision with any other frame}] = \lambda T_t e^{-2\lambda T_t}$$

In UW networks, on the other hand, due to appreciable signal propagation delay T_p compared

to T_t, the collision vulnerability window is larger than $2T_t$. Particularly, it can be observed from Fig. 1(b) that, an additional frame generated at a neighboring transmitter at time t', where $t + T_p - T_t - T_p^{\max} \leq t' \leq t + T_p + T_t$, may lead to collision with a frame that is being received at the receiver in question from time $t + T_p$. That is, irrespective of the receiver's distance from its transmitter, traffic generated from the neighboring transmitters within a time window $2T_t + T_p^{\max}$ can cause collision at the receiver, where $T_p^{\max} = \frac{R}{v}$ is the maximum propagation delay up to a node's communication range R, and v is the underwater acoustic signal propagation speed.

However, unlike in short-range RF networks, only some of the neighborhood generated frames in the interval $2T_t + T_p^{\max}$ will lead to collision with a frame that is currently being received. Specifically, referring to Fig. 1(b), a collision with the frame currently being received at time $t + T_p$ occurs if the frame generation instant t' at a neighboring transmitter and the associated propagation delay T_p' up to the receiver in question satisfy either of the two conditions in (1).

$$t + T_p < t' + T_p' < t + T_p + T_t \tag{1a}$$

$$\text{or, } t + T_p < t' + T_p' + T_t < t + T_p + T_t \tag{1b}$$

Looking from the receiver's perspective, as long as its frame reception duration does not overlap with any other frame arrivals from its neighbors, the frame will be successful. Thus, a frame of size T_t, whose reception starts at time $t + T_p$, will be successful if no additional arrival at the receiver occurs during the interval $2T_t$ (from $t + T_p - T_t$ to $t + T_p + T_t$), even though the possible arrivals during this time could be caused by the generation process over a larger time duration (which is $2T_t + T_p^{\max}$ in case of UWN). This concept is further pictorially depicted in Fig. 2, where

Figure 2: Pictorial representation of collision vulnerability concept.

the duration $|t_1 - t_3|$ is the frame generation window causing possible collision vulnerability, and $|t_2 - t_3|$ is the vulnerability window with respect to the reception process.

Consider the number of frames that arrive in window $[t_2, t_3] = m$, and the generated ones

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during $[t_1, t_3] = n$. Irrespective of the signal propagation delay, we have [14, Ch. 3]

$$\Pr\left[m \text{ out of } n \text{ frames arrive during } [t_2, t_3]\right] \triangleq P_n(m) = \binom{n}{m} p^m (1-p)^{n-m} \tag{2}$$

where $p = \frac{|t_2 - t_3|}{|t_1 - t_3|} = \frac{2T_t}{|t_1 - t_3|}$. Since the frame generation process in the system is Poisson, the arrivals in the window $[t_2, t_3]$ can also be approximated as Poisson distributed, as follows. The frame arrival rate in the system is $\lambda = \frac{n}{|t_1 - t_3|}$. In case of a homogeneous frame generation process, the window $|t_1 - t_3|$ can be increased arbitrarily, leading to $n \to \infty$ and $p \to 0$, keeping the product $np = 2\lambda T_t$ a constant. Hence, (2) can be approximated as:

$$P_n(m) \approx e^{-np} \frac{(np)^m}{m!} = e^{-2\lambda T_t} \frac{(2\lambda T_t)^m}{m!} \tag{3}$$

The frame success probability is, $P_n(0) = e^{-2\lambda T_t}$. Therefore, the normalized system throughput of Aloha-uw with fixed frame size is given by:

$$\eta_{\text{Aloha-uw}}^{\text{(fixed)}} = \lambda T_t e^{-2\lambda T_t} \tag{4}$$

which is the same as the Aloha-rf throughput, and is valid for any propagation delay.

3.2. Aloha-uw with variable frame size

Normalized system throughput in Aloha-rf with Poisson distributed arrival process and variable (exponentially distributed) frame size can be found as [15, Ch. 3]:

$$\eta_{\text{Aloha-RF}}^{\text{(exp)}} = \lambda T_t \cdot \Pr[\text{system idle at the frame arrival instant}]$$
$$\cdot \Pr[\text{next interarrival time } \tau > \text{current frame duration } \mathbf{T_t}]$$
$$= \lambda T_t e^{-\lambda T_t} \cdot \int_0^\infty \Pr[\tau > t | \mathbf{T_t} = t] \cdot \Pr[\mathbf{T_t} = t]$$
$$= \lambda T_t e^{-\lambda T_t} \cdot \int_0^\infty e^{-\lambda t} \frac{1}{T_t} e^{-\frac{t}{T_t}} dt = \frac{\lambda T_t}{1 + \lambda T_t} e^{-\lambda T_t} \tag{5}$$

Following a similar logic as in the case of Aloha-uw with fixed frame size, irrespective of the signal propagation delay, the normalized system throughput $\eta_{\text{Aloha-uw}}^{\text{(exp)}}$ is also given by (5).

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4. Slotted-Aloha in UWN

First, irrespective of the nodal coverage range in a short-range RF network, we have the throughput expression for S-Aloha-rf as [15, Ch. 4]:

$$\eta_{\text{S-Aloha-rf}} = \lambda T_t e^{-\lambda T_t} \tag{6}$$

In a UWN with randomly located nodes, propagation delay T_p of a frame to the receiver varies between 0 and T_p^{max} (see Fig. 3(b) and (c)). Since the synchronization in a slotted access protocol is done at the transmitter nodes, to resemble the one-slot S-Aloha vulnerability concept as in short-range RF communications, i.e., to ensure that a frame collision probability in S-Aloha-uw is only due to non-zero additional arrivals in one slot, a buffer time T_p^{max} is needed to accommodate the arrival uncertainty due to propagation delay. Thus, unlike in S-Aloha-rf, where the slot size is $T_s^{\text{rf}} = T_t$ (see Fig. 3(a)), the slot size in S-Aloha-uw should be $T_s^{\text{uw}} = T_t + T_p^{\text{max}} \triangleq T_{s1}$ (see Fig. 3(c)). Note from Fig. 3(b) that, a slot size $T_{s1} = T_t + T_p^{\text{max}}$ ensures that the frames generated in
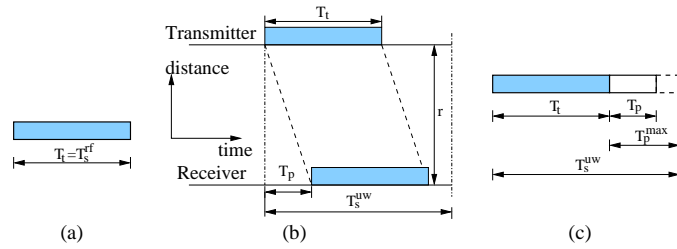

Figure 3: Slotting concepts. (a) Slot size in S-Aloha-rf; (b) effect of signal propagation delay on the time lag between a frame transmission and its reception process; (c) slot size in S-Aloha-uw.

a slot do not collide with the ones generated in another slot. Also, S-Aloha-rf like frame success probability is achieved as long as $T_p^{\text{max}} < T_t$. However, if $T_p^{\text{max}} \geq T_t$, more than one frame generated in a slot do not necessarily cause a frame collision at the receiver. So, the S-Aloha-uw frame throughput for ad hoc networks has to be computed differently for the two regimes of propagation delay.

Case 1: $T_p^{\text{max}} < T_t$

The throughput computation in this regime is done similarly as in S-Aloha-rf. Thus, the normalized

system throughput in S-Aloha-uw for $T_p^{\max} < T_t$ is given by:

$$\eta_{\text{S-Aloha-uw}}(T_p^{\max} < T_t) = \lambda T_t \cdot \Pr[\text{no additional arrival in a slot}] = \lambda T_t e^{-\lambda\left(T_t + T_p^{\max}\right)} \quad (7)$$

Case 2: $T_p^{\max} \geq T_t$

Let the receiver's distance from its intended transmitter be $\mathbf{r} = r$. The frame success probability P_S of S-Aloha-uw is obtained from the conditional success probabilities as:

$$P_S = \int_{r=0}^{R} \Pr[\text{success}|\mathbf{r} = r] \cdot p(r) \quad (8)$$

where $p(r) \overset{\Delta}{=} \Pr[\mathbf{r} = r] = \Pr[\text{intended transmitter's distance to the receiver}, \mathbf{r} = r]$. In a network with uniformly random distributed nodes, if a transmitter-receiver pair is chosen independent of the distance between them, considering the receiver is at the center of its circular communication range, a transmitter can be located at any point in the circular region. Then, the density function (pdf) of the distance $\mathbf{r}$ between a transmitter and the receiver is:

$$f_{\mathbf{r}}(r) = \begin{cases} \frac{2r}{R^2}, & 0 \leq r \leq R \\ 0, & \text{elsewhere} \end{cases} \quad (9)$$

Hence,

$$p(r) \equiv \Pr[r \leq \mathbf{r} \leq r + dr] = f_{\mathbf{r}}(r)dr = \frac{2r\,dr}{R^2} \quad (10)$$

The regime of $T_p^{\max} \geq T_t$ is further divided into two: (a) $T_t \leq T_p^{\max} \leq 2T_t$, and (b) $T_p^{\max} > 2T_t$.

Case 2-a: $T_t \leq T_p^{\max} \leq 2T_t$

With R denoting the nodal communication range and v denoting the acoustic signal propagation speed, we have the following three sub-regions of $\mathbf{r}$ for which the conditional frame success probabilities are calculated separately as follows:

In *sub-region 1*, where $0 \leq \mathbf{r} \leq R - T_t v$, if n additional frames are generated in the same slot along with the intended frame, the frame will still be successful as long as all n other frames have propagation delay $\mathbf{T_p'} \geq \frac{r}{v} + T_t$. Since $\mathbf{T_p'} = \frac{r'}{v}$, where $\mathbf{r'}$ is a random variable (RV) representing

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the distance of a neighboring transmitter from the intended receiver, the above condition reduces to $\mathbf{r}' \geq r + T_t v$. Accordingly, the conditional frame success probability is given by

$$
\begin{aligned}
P_{S_{1a}} &= \sum_{n=0}^{\infty} \left(\Pr[\mathbf{r}' \geq r + T_t v] \right)^n \cdot \Pr[n \text{ additional arrivals in a slot}] \\
&= \sum_{n=0}^{\infty} \left[1 - \left(\frac{r + T_t v}{R} \right)^2 \right]^n \frac{(\lambda T_{s1})^n}{n!} e^{-\lambda T_{s1}} = e^{-\lambda T_{s1} \left(\frac{r + T_t v}{R} \right)^2}
\end{aligned}
\tag{11}
$$

where, from (9),

$$
\Pr[\mathbf{r}' \leq r + T_t v] = \begin{cases} \frac{(r + T_t v)^2}{R^2}, & 0 \leq r \leq R - T_t v \\ 1, & R - T_t v \leq r \leq R \\ 0, & \text{elsewhere} \end{cases}
\tag{12}
$$

In *sub-region 2*, where $R - T_t v < \mathbf{r} < T_t v$, the frame will be successful if there are no additional frames generated from any neighboring transmitters in the same slot. Accordingly,

$$
P_{S_{2a}} = e^{-\lambda T_{s1}}
\tag{13}
$$

Similarly as in sub-region 1, in *sub-region 3*, where $T_t v \leq \mathbf{r} \leq R$, the intended frame will be successful if there are n additional frames in the same slot generated at a distance $\mathbf{r}''$ such that $0 \leq \mathbf{r}'' \leq r - T_t v$. The conditional frame success probability is given by:

$$
\begin{aligned}
P_{S_{3a}} &= \sum_{n=0}^{\infty} \left(\Pr[0 \leq \mathbf{r}'' \leq r - T_t v] \right)^n \frac{(\lambda T_{s1})^n}{n!} e^{-\lambda T_{s1}} \\
&= \sum_{n=0}^{\infty} \left(\frac{r - T_t v}{R} \right)^{2n} \frac{(\lambda T_{s1})^n}{n!} e^{-\lambda T_{s1}} = e^{-\lambda T_{s1} \left[1 - \left(\frac{r - T_t v}{R} \right)^2 \right]}
\end{aligned}
\tag{14}
$$

where

$$
\Pr[0 \leq \mathbf{r}'' \leq r - T_t v] = \begin{cases} \frac{(r - T_t v)^2}{R^2}, & T_t v \leq r \leq R \\ 0, & 0 \leq r \leq T_t v \\ 1, & \text{elsewhere} \end{cases}
\tag{15}
$$

Using (8), the net frame success probability $P_{S_a}(T_t \leq T_p^{\max} \leq 2T_t)$ is obtained as:

$$
P_{S_a} = \int_{r=0}^{R - T_t v} P_{S_{1a}} \cdot p(r) + \int_{R - T_t v}^{T_t v} P_{S_{2a}} \cdot p(r) + \int_{T_t v}^{R} P_{S_{3a}} \cdot p(r)
\tag{16}
$$

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Hence, the normalized system throughput is:

$$\eta_{\text{S-Aloha-uw}}(T_t \leq T_p^{\max} \leq 2T_t) = \lambda T_t P_{S_a} \tag{17}$$

Case 2-b: $T_p^{\max} > 2T_t$

In this case, *sub-region 1* is $0 \leq \mathbf{r} \leq T_t v$, where the success probability $P_{S_{1b}}$ is given by (11).

In *sub-region 2*, $T_t v < \mathbf{r} < R - T_t v$. If n additional frames from neighboring transmitters are generated, of which n' are from a distance $\mathbf{r}'$ such that $R \geq \mathbf{r}' \geq r + T_t v$ and $n - n'$ are from a distance $\mathbf{r}''$ such that $0 \leq \mathbf{r}'' \leq r - T_t v$, the intended frame to the receiver will still be successful. Thus, the frame success probability $P_{S_{2b}}$ is given by

$$
\begin{aligned}
P_{S_{2b}} &= \sum_{n=0}^{\infty} \sum_{n'=0}^{n} \left(\Pr[R \geq \mathbf{r}' \geq r + T_t v] \right)^{n'} \left(\Pr[0 \leq \mathbf{r}'' \leq r - T_t v] \right)^{n-n'} \frac{(\lambda T_{s1})^n}{n!} e^{-\lambda T_{s1}} \\
&= \sum_{n=0}^{\infty} \sum_{n'=0}^{n} \left[1 - \left(\frac{r + T_t v}{R} \right)^2 \right]^{n'} \left(\frac{r - T_t v}{R} \right)^{2(n-n')} \frac{(\lambda T_{s1})^n}{n!} e^{-\lambda T_{s1}}
\end{aligned} \tag{18}
$$

The *sub-region 3* is $R - T_t v \leq \mathbf{r} \leq R$, where the success probability $P_{S_{3b}}$ is given by (14).

Combining, the unconditional success probability is given by

$$P_{S_b} = \int_{r=0}^{T_t v} P_{S_{1b}} \cdot p(r) + \int_{r=T_t v}^{R - T_t v} P_{S_{2b}} \cdot p(r) + \int_{r=R - T_t v}^{R} P_{S_{3b}} \cdot p(r) \tag{19}$$

Hence, the normalized system throughput is obtained as:

$$\eta_{\text{S-Aloha-uw}}(T_p^{\max} > 2T_t) = \lambda T_t P_{S_b} \tag{20}$$

5. A modified S-Aloha for UWN

From the analysis in Section 4 it can be noted that, with the naive slotting concept in S-Aloha-uw, the slot size has to be larger than that in S-Aloha-rf by $T_p^{\max} = \frac{R}{v}$, in anticipation that a transmitter-receiver pair can be up to R distance apart. However, as depicted in Fig. 3(b) and (c), in most cases transmitter-receiver pairs are less than R distance apart, and so a reception process is completed before the S-Aloha-uw slot ends. Note that, in one-to-one communication, after the frame reception at a node is completed, the system remains idle for the duration $T_p^{\max} - T_p$, thereby causing reduction in system throughput. It is also clear from (7) that, with $T_p^{\max} < T_t$ and

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for a given λ, the higher the ratio $\frac{T_p^{\max}}{T_t}$, the lesser the system throughput $\eta_{\text{S-Aloha-uw}}$ compared to $\eta_{\text{S-Aloha-rf}}$ in (6). Similar trends are expected at $T_p^{\max} \geq T_t$ (see (17) and (20)), which are presented in Section 6. In ad hoc network multiaccess communication, other than having reduced system throughput, no additional intuition is derived from the cases of $T_p^{\max} \geq T_t$. Therefore, we restrict our further studies on S-Aloha-uw to $T_p^{\max} < T_t$.

Since it is likely that almost in all cases $r < R$, it may be wise to reduce the slot size appropriately, such that the frames in most cases are successful, while in some cases they may collide with the preceding or/and subsequent frames. An optimally chosen slot size would minimize the system idling time without increasing the collision vulnerability, so as to increase the overall system throughput. We call this modified slotted-Aloha protocol as *mS-Aloha-uw*. The modified slotting
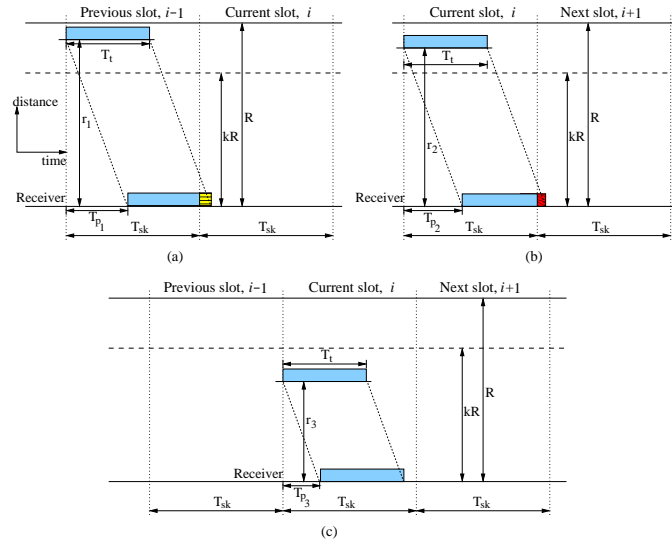

Figure 4: Modified slotting concept in UWN. $T_{sk} = T_t + kT_p^{\max}$, where $0 \leq k \leq 1$ and $T_p^{\max} < T_t$. (a) A frame from a r_1 distance away transmitter scheduled in the previous slot may cause collision with a frame in the current slot if $kR < r_1 < R$. (b) A frame in the current slot from a r_2 distance away transmitter may encounter collision with a frame scheduled in the next slot if $kR < r_2 < R$. (c) A frame scheduled from a r_3 distance away transmitter, $0 < r_1 < kR$, does not cause collision with the frames in other slots.

concept with $T_p^{\max} < T_t$ is shown in Fig. 4. In this approach, the buffer time to accommodate the transmitter-to-receiver propagation delay is reduced to $kT_p^{\max}$, where k is termed as the *slot*

size reduction factor. Since $0 \leq k \leq 1$, we have $kT_p^{\mathbf{max}} \leq T_p^{\mathbf{max}}$, and hence the modified total slot size $T_{sk} = T_t + kT_p^{\mathbf{max}} \leq T_{s1}$. Note that, $k = 0$ corresponds to the slot size in S-Aloha-rf, but it will introduce frame vulnerability in S-Aloha-uw from the previous slot as well as the next slot; whereas $k = 1$ corresponds to the naive S-Aloha-uw, in which case there would not be any collision with frames from any other slots.

The throughput of mS-Aloha-uw can be computed using the general expression for the success probability P_S given in (8), where the RV $\mathbf{r}$ (now identified as $\mathbf{r_i}$) is the distance of the transmitter that has a frame scheduled in the current slot (slot i) to an intended receiver. However, in addition to the collision probability due to more than one frame scheduled in slot i (i.e., more than one arrivals in slot $i - 1$), depending on the value of k, two conditions for a frame collision exist. For $k \leq 0.5$, a frame transmitted in slot i can be vulnerable simultaneously due to the neighboring nodes' transmissions in the two adjacent slots $i - 1$ and $i + 1$; whereas, for $k \geq 0.5$, vulnerability of a frame can be caused by a transmission in either the previous slot $i - 1$ or the next slot $i + 1$. Accordingly, the successful reception probability of a frame is computed differently for $0 \leq k \leq 0.5$ and $0.5 \leq k \leq 1.0$. In each of these two cases, the frame success probability varies at different windows of $\mathbf{r_i} = r$. For example, with $k \leq 0.5$, at $\mathbf{r_i} = r$, $0 \leq r \leq kR$, a frame reception beginning in slot i is successful if there is only one frame scheduled in slot i, and there are possibly n_p neighboring frame transmissions scheduled in slot $i - 1$ but all of them have propagation delay $\mathbf{T_p^{(P_j)}} = T_p^{(p_j)}$ to the receiver in question such that $\mathbf{T_p^{(P_j)}} \leq \frac{r+kR}{v}$, $\forall\, j \leq n_p$. Since $\mathbf{T_p^{(P_j)}} = \frac{\mathbf{r_p}}{v}$, the above condition reduces to $\mathbf{r_p} \leq r + kR$, where $\mathbf{r_p}$ is an i.i.d. RV representing the distance of the receiver in question from a neighboring transmitter that has a frame scheduled in slot $i - 1$. So, the probability that the current frame does not collide with the one scheduled in slot $i - 1$ is, $\Pr\big[\mathbf{r_p} \leq r + kR\big]$, given by (12) with $T_t v$ replaced by kR.

Likewise, the condition for no collision with a frame in the next slot (slot $i + 1$) is: $\mathbf{T_p^{(n_j)}} \geq \frac{r-kR}{v}$, $\forall\, j \leq n_n$, where $T_p^{(n_j)}$ is the propagation delay up to the receiver from the j-th neighboring transmitter with a scheduled frame in slot $i + 1$. For an appreciable (non-zero) value of $T_p^{(n_j)}$, the probability of no collision with a frame scheduled in slot $i + 1$ becomes: $\Pr\big[\mathbf{r_n} \geq r - kR\big]$, where $\mathbf{r_n}$ is an i.i.d. RV representing the receiver's distance from the neighboring transmitter.

$\Pr\big[\mathbf{r_n} \geq r - kR\big] = 1 - \Pr\big[\mathbf{r_n} \leq r - kR\big]$ is obtained from (15) with $T_t v$ replaced by kR.

Considering all values of r in $(0, R)$, the frame success probability for the two regimes of k is obtained below.

Case 1: $0 \leq k \leq 0.5$

In this range, the frame success probability is given by (21),

$$
\begin{aligned}
P_S(0 \leq k \leq 0.5) = {} & \int_{r=0}^{kR} p(0^{(i)}) \sum_{n_p=0}^{\infty} p(n_p^{(i-1)}) \Big(\Pr[\mathbf{r_p} \leq r + kR]\Big)^{n_p} p(r) \\
& + \int_{kR}^{R-kR} p(0^{(i)}) \sum_{n_p=0}^{\infty} p(n_p^{(i-1)}) \Big(\Pr[\mathbf{r_p} \leq r + kR]\Big)^{n_p} \sum_{n_n=0}^{\infty} p(n_n^{(i+1)}) \Big(\Pr[\mathbf{r_n} \geq r - kR]\Big)^{n_n} p(r) \\
& + \int_{R-kR}^{R} p(0^{(i)}) \sum_{n_n=0}^{\infty} p(n_n^{(i+1)}) \Big(\Pr[\mathbf{r_n} \geq r - kR]\Big)^{n_n} p(r) \\
\triangleq {} & P'_{S_1} + P'_{S_2} + P'_{S_3}
\end{aligned}
\tag{21}
$$

where $p(f^{(j)}) \triangleq \Pr[f$ frames scheduled in slot $j]$, and $p(r)$ is defined in (10). Note that, in addition to accounting the possibility of more than one arrival in the current slot, P'_{S_1} captures the frame vulnerability due to arrivals in the previous slot, P'_{S_2} absorbs the vulnerability due to arrivals in the previous slot as well as the next slot, whereas P'_{S_3} accommodates the vulnerability due to arrivals in the next slot.

With the assumption of Poisson distributed traffic arrival process at a rate λ, and using (10) and (12), the expression for P'_{S_1} in (21) is obtained as:

$$
P'_{S_1} = \int_{r=0}^{kR} e^{-\lambda T_{sk}} \sum_{n_p=0}^{\infty} \frac{(\lambda T_{sk})^{n_p} e^{-\lambda T_{sk}}}{n_p!} \left(\frac{r + kR}{R}\right)^{2n_p} \frac{2r\,dr}{R^2}
\tag{22a}
$$

$$
\begin{aligned}
= {} & \frac{e^{-2\lambda T_{sk}}}{\lambda T_{sk}} \left[e^{4\lambda T_{sk} k^2} - e^{\lambda T_{sk} k^2} \right] \\
& - \frac{2k e^{-2\lambda T_{sk}}}{\sqrt{\lambda T_{sk}}} \left[e^{4\lambda T_{sk} k^2} D_+\left(2k\sqrt{\lambda T_{sk}}\right) - e^{\lambda T_{sk} k^2} D_+\left(k\sqrt{\lambda T_{sk}}\right) \right]
\end{aligned}
\tag{22b}
$$

where $D_+(x) = e^{-x^2} \int_0^x e^{t^2} dt$ is the Dawson's integral [16, Ch. 7].

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Using (10), (12), and (15), P'_{S_2} is obtained as:

$$
P'_{S_2} = \int_{kR}^{R-kR} e^{-\lambda T_{sk}} \sum_{n_p=0}^{\infty} \frac{(\lambda T_{sk})^{n_p} e^{-\lambda T_{sk}}}{n_p!} \left(\frac{r+kR}{R}\right)^{2n_p}
$$

$$
\cdot \sum_{n_n=0}^{\infty} \frac{(\lambda T_{sk})^{n_n} e^{-\lambda T_{sk}}}{n_n!} \left[1 - \frac{(r-kR)^2}{R^2}\right]^{n_n} \frac{2rdr}{R^2} \tag{23a}
$$

$$
= \frac{2e^{-2\lambda T_{sk}}}{(4\lambda T_{sk}k)^2} \left[\left(4\lambda T_{sk}k(1-k)-1\right)e^{4\lambda T_{sk}k(1-k)} \left(4\lambda T_{sk}k^2-1\right)e^{4\lambda T_{sk}k^2}\right] \tag{23b}
$$

Similarly, the expression for P'_{S_3} is given by:

$$
P'_{S_3} = \int_{r=R-kR}^{R} e^{-\lambda T_{sk}} \sum_{n_n=0}^{\infty} \frac{(\lambda T_{sk})^{n_n} e^{-\lambda T_{sk}}}{n_n!} \left[1 - \frac{(r-kR)^2}{R^2}\right]^{n_n} \frac{2rdr}{R^2} \tag{24a}
$$

$$
= \frac{e^{-\lambda T_{sk}}}{\lambda T_{sk}} \left[e^{-\lambda T_{sk}(1-2k)^2} - e^{-\lambda T_{sk}(1-k)^2}\right]
$$

$$
+ \frac{\sqrt{\pi}ke^{-\lambda T_{sk}}}{\sqrt{\lambda T_{sk}}} \left[\text{erf}\left(\sqrt{\lambda T_{sk}}(1-k)\right) - \text{erf}\left(\sqrt{\lambda T_{sk}}(1-2k)\right)\right] \tag{24b}
$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$.

Using (22b), (23b), and (24b), the normalized system throughput for $0 \leq k \leq 0.5$ is found as:

$$
\eta_{\text{mS-Aloha-uw}}(0 \leq k \leq 0.5) = \lambda T_t(P'_{S_1} + P'_{S_2} + P'_{S_3}) \tag{25}
$$

At a limiting case $k \to 0$, the integrations in (22a) and (24a) vanish, and thus, from (23a) and (25) the normalized system throughput is reduced to:

$$
\lim_{k \to 0} \eta_{\text{mS-Aloha-uw}} \triangleq \eta^0_{\text{mS-Aloha-uw}} = \lambda T_t e^{-2\lambda T_t} \tag{26}
$$

which is the same as the Aloha throughput with fixed frame transmission time T_t, given in (4).

Case 2: $0.5 \leq k \leq 1.0$

The frame success probability in this case is given by (27).

$$
P_S(0.5 \leq k \leq 1.0) = \int_{r=0}^{R-kR} p(0^{(i)}) \sum_{n_p=0}^{\infty} p(n_p^{(i-1)}) \left(\text{Pr}[\mathbf{r_p} \leq r+kR]\right)^{n_p} p(r)
$$

$$
+ \int_{r=R-kR}^{kR} p(0^{(i)})p(r) + \int_{r=kR}^{R} p(0^{(i)}) \sum_{n_n=0}^{\infty} p(n_n^{(i+1)}) \left(\text{Pr}[\mathbf{r_n} \geq r-kR]\right)^{n_n} p(r)
$$

$$
\triangleq P''_{s_1} + P''_{s_2} + P''_{s_3} \tag{27}
$$

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Note that, unlike in (21), P''_{s_2} in (27) is represents the vulnerability due to additional arrivals in the current slot (i.e., slot i) only.

Using (12) and (15), similarly as in (22), (23), and (24), we have the expressions for P''_{s_1}, P''_{s_2}, and P''_{s_3}:

$$P''_{s_1} = \int_{r=0}^{R-kR} e^{-2\lambda T_{sk}} \cdot e^{\lambda T_{sk} \left(\frac{r+kR}{R}\right)^2} \cdot \frac{2r\,dr}{R^2} \tag{28a}$$

$$= \frac{e^{-2\lambda T_{sk}}}{\lambda T_{sk}} \left[e^{\lambda T_{sk}} - e^{\lambda T_{sk} k^2} \right] - \frac{2k}{\sqrt{\lambda T_{sk}}} \left[e^{-\lambda T_{sk}} D_+ \left(\sqrt{\lambda T_{sk}}\right) - e^{-\lambda T_{sk}(2-k^2)} D_+ \left(k\sqrt{\lambda T_{sk}}\right) \right] \tag{28b}$$

$$P''_{s_2} = \int_{r=R-kR}^{kR} e^{-\lambda T_{sk}} \cdot \frac{2r\,dr}{R^2} = e^{-\lambda T_{sk}}(2k-1) \tag{29}$$

$$P''_{s_3} = \int_{r=kR}^{R} e^{-\lambda T_{sk}} \cdot e^{-\lambda T_{sk} \left(\frac{r-kR}{R}\right)^2} \cdot \frac{2r\,dr}{R^2} \tag{30a}$$

$$= e^{-\lambda T_{sk}} \left[\frac{1 - e^{-\lambda T_{sk}(1-k)^2}}{\lambda T_{sk}} + k\sqrt{\frac{\pi}{\lambda T_{sk}}} \mathrm{erf}\left(\sqrt{\lambda T_{sk}}(1-k)\right) \right] \tag{30b}$$

The corresponding normalized system throughput is:

$$\eta_{\text{mS-Aloha-uw}}(0.5 \le k \le 1.0) = \lambda T_t \cdot (P''_{s_1} + P''_{s_2} + P''_{s_3}) \tag{31}$$

Again, in the limit $k \to 1$, the integrations in (28a) and (30a) reduce to 0, and hence, from (29) and (31) the normalized system throughput becomes:

$$\lim_{k \to 1} \eta_{\text{mS-Aloha-uw}} \triangleq \eta^1_{\text{mS-Aloha-uw}} = \lambda T_t e^{-\lambda\left(T_t + T_p^{\max}\right)} \tag{32}$$

which is the same as the naive S-Aloha-uw throughput performance given in (7).

5.1. Validity of the analysis in short distance RF wireless environment

Let us now check how the mS-Aloha-uw throughput analysis applies to the S-Aloha-rf case.

Since in RF wireless communication the signal propagation speed $c \gg$ underwater acoustic signal propagation speed v, the slot size is $T_{sk} = T_t + \frac{kR}{c} \to T_t$, for any value of k.

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Also, the propagation delay associated with a transmitter-receiver distance r is $\frac{r}{c} \to 0$. Accordingly, the condition $\mathbf{T_p^{(p_j)}} \leq \frac{r+kR}{v}$ in a short distance RF wireless communication becomes $\mathbf{T_p^{(p_j)}} \to 0$, $\forall j \leq n_p$, and hence, $\Pr[\mathbf{T_p} \leq \frac{r+kR}{c}] \to 1$, which implies, $\Pr[\mathbf{r_p} \leq r + kR]$ can be replaced by 1. Likewise, since $T_p^{(n_j)} \to 0$, $\forall j \leq n_n$, for any value of k and r, $\Pr[\mathbf{r_n} \geq r - kR]$ can be replaced by 1.

With the above reduced expressions, irrespective of the value of k, from (8), the probability of a frame success in any slot i is given by,

$$P_S = \int_{r=0}^{R} \Pr[\text{no other frame scheduled in slot } i] \cdot \Pr[\mathbf{r_i} = r] = \int_{r=0}^{R} e^{-\lambda T_t} \frac{2r\,dr}{R^2} = e^{-\lambda T_t} \quad (33)$$

which leads to the same normalized throughput expression as in (6).

In the following Section, relative throughput performance results are discussed.

6. Results and Discussion

System throughput performance of the Aloha variants in UW as well as RF wireless networks have been studied in MATLAB using the analytic expressions developed in Sections 3, 4, and 5, and via C based discrete event simulations of a random network. We have not used a standard network simulator in this study for the following reasons: (a) Our current study has been rather focussed on MAC layer only; it does not involve multiple layers or the system as a whole. Also, as it is apparent from the analytic proofs, the isolated underwater MAC layer problem itself is quite involved. (b) While the basic underwater MAC characterization remains the same, we anticipate that, besides the propagation speed, there might be some impact of channel characteristics on the multiaccess performance, which could be tested using a standard network simulator. However, the standard simulators do not have a practical underwater channel model built in. In fact, to the best of our knowledge, a thorough characterization of (variability of) underwater wireless channel for network application is yet to be available. Therefore, our underwater network simulation studies focussed on the effect of random propagation delay in addition to the random packet arrival process on the system performance.

In the numerical computations and network simulation studies, following the underwater modem specifications [1], the channel rate was considered $R_c = 16$ kbps. Acoustic signal speed is

$v = 1500$ m/s. The nodes were assumed to have homogeneous circular communication range, and they can have any-to-any communication. Default value of communication range was $R = 20$ m. Since the RTS frame size is 36 Bytes (as in 802.11b/g standard without interoperability) or 44 Bytes (as in 802.11b/g standard with interoperability), the default frame size was taken as an average, $F = 40$ Bytes. Also, as allowed in standard sensor motes (e.g., Crossbow MICA2 motes), the largest frame size taken was $F = 240$ Bytes. For S-Aloha, the value of R was chosen appropriately to ensure if the maximum internodal signal propagation delay $T_p^{\max} = \frac{R}{v}$ is less than, or equal to, or greater than the frame transmission time $T_t = \frac{F}{R_c}$.

In the simulation, to study the effect of frame collisions at a receiving node, $N = 200$ randomly located nodes were taken around the receiving node's communication range. In each iteration, a randomly located transmitter was chosen, and the other neighboring transmitters' activities were controlled by varying the (Poisson distributed) frame arrival rate λ_0 at a node, where λ_0 and the system-wide arrival rate λ are related as $\lambda = N\lambda_0$. These neighboring transmitters may send data to their chosen respective receivers. To compute the throughput performance, for every desired frame reception, we checked for any possible time overlap with the frames that may have been generated from the neighboring transmitters. For each set of parameters, average performance was computed over 5000 iterations to obtain sufficiently high confidence over the simulated data.

In Fig. 5, throughput performances of Aloha and S-Aloha are compared when applied in short-range RF networks and UW networks, respectively, with constant sized frames. The analytic observations match very well with the simulation results. Matched results of Aloha-uw and Aloha-rf confirm that the signal propagation speed does not have an effect on the Aloha throughput performance. However, the sensitivity of propagation delay in S-Aloha quite apparent, as S-Aloha-uw performs poorer compared to the S-Aloha-rf. The performance degradation is more prominent because of the chosen high $T_p^{\max}$ (13.3 ms), which is comparable to the value of T_t (20 ms). Note, the analysis indicates that, under the condition $T_t > T_p^{\max}$, the S-Aloha-uw throughput performance would be in between S-Aloha-rf and Aloha-rf.

The Aloha-uw performance with variable (exponentially distributed) frame size is shown in Fig. 6, which further verifies the lack of sensitivity of signal propagation delay on pure Aloha
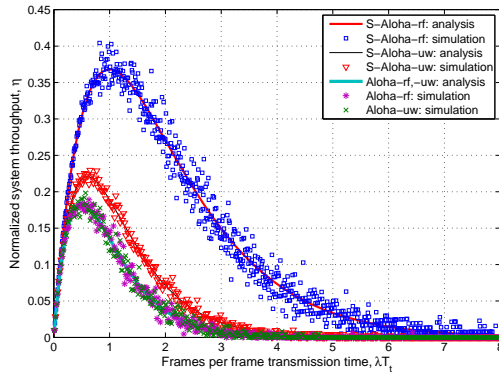
Figure 5: Performance comparison of Aloha and S-Aloha with fixed frame size. $F = 40$ Bytes, $R = 20$ m.
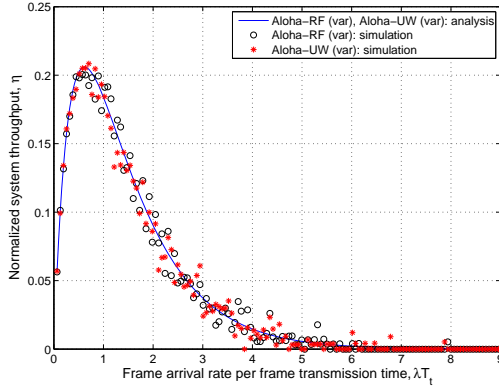

Figure 6: Performance comparison of Aloha-uw and Aloha-rf with exponentially distributed frame size. Average frame size $F = 128$ Bytes.

performance.

The dependence of the ratio $\frac{T_p^{\max}}{T_t}$ on S-Aloha-uw performance is apparent from the simulated maximum throughput results in Fig. 7, where the communication range is kept fixed, but the frame size is varied. While the Aloha and S-Aloha-rf performances are fairly constant (nearly 0.184 and 0.368, respectively), S-Aloha-uw performance improves as the transmission time increasingly dominates over the propagation time. This is because, relatively less propagation delay implies lesser system idling time in S-Aloha-uw.

Fig. 8 further shows the nature of variation of maximum system throughput for different values
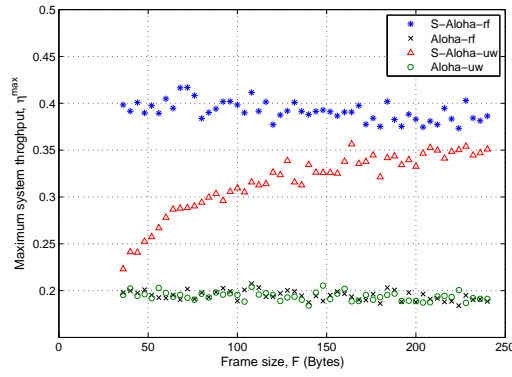
Figure 7: Variation of maximum throughput with frame size. $R = 20$ m.

of $\frac{T_p^{\max}}{T_t}$, in which our particular interest is the region where $\frac{T_p^{\max}}{T_t} > 1$. First, note that the
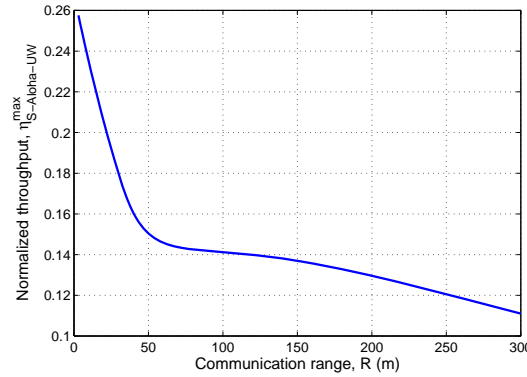

Figure 8: Variation of maximum throughput with internodal distance. $F = 40$ Bytes, $\lambda T_t = 0.4$.

maximum throughput is monotonically decreasing as $T_p^{\max}$ increases. This observation prompted us to restrict our mS-Aloha-uw studies to $\frac{T_p^{\max}}{T_t} \leq 1$, beyond which the performance of simple Aloha will always be better. Second, the rate of decrease in maximum throughput is not sharp after $\frac{T_p^{\max}}{T_t} = 1$, which is because, beyond this value there is a finite probability of receiving a frame correctly even though there could be more than one transmissions within the coverage range of a receiver.

Throughput performance of mS-Aloha-uw at different values of slot reduction factor k are shown in Fig. 9. The plots indicate that, by choosing properly reduced slot size (via controlling

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k) the underwater S-Aloha performance can be significantly improved. Note that $k = 0$ implies the slot size $T_{sk} = T_t$, and it gives the same throughput performance as in Aloha. This is because,
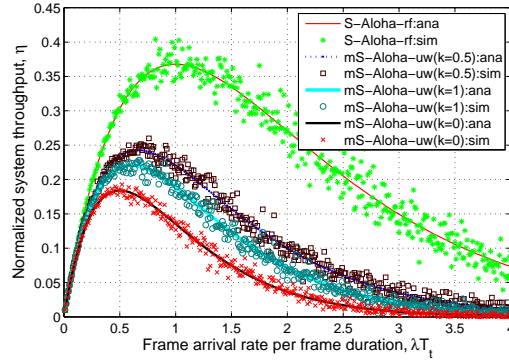

Figure 9: mS-Aloha-uw performance at different values of k, and comparison with S-Aloha-rf protocol. $\frac{T_p^{\max}}{T_t} = \frac{2}{3}$

having no buffer time, a frame reception vulnerability duration becomes $2T_t$ (as in Aloha), and it can collide with the a frame in the preceding slot and/or the next slot. At the other extreme, with very high buffer time the spill over duration of an arriving frame beyond the slot boundary is minimized. But most of the time the frame arrivals to the receivers are completed well within the slot time, thus leaving much room to system idling. The S-Aloha-rf throughput plot on the same graph also indicates that, due to added randomness in frame arrival process in acoustic wireless networks, mS-Aloha-uw performance is quite poorer, and the arrival rate corresponding to the peak performance of mS-Aloha-uw tends to that of Aloha. A good match of the analytically obtained plots with the simulated results also verify correctness of the analysis. In the subsequent discussion, we present some analytic plots to show the conditions for maximum system throughput.

The dependence of the maximum throughput performance on slot size reduction factor k is shown in Fig. 10, where $\frac{T_p^{\max}}{T_t}$ (obtained by choosing suitable R) is taken as the parameter. Observe that, for a given $T_p^{\max}$ (i.e., for a given communication range R), there is an optimum k that offers the maximum system throughput. At $T_p^{\max} = T_t$, $\eta_{\text{mS-Aloha-uw}}^{\max} = 0.2157$ (achieved at $k = 0.52$), whereas $\eta_{\text{S-Aloha-uw}}^{\max} = 0.1839$ (when $k = 1$). Hence, mS-Aloha-uw offers a 17.3% gain in maximum throughput at $T_p^{\max} = T_t$ by optimally choosing k, where the percentage
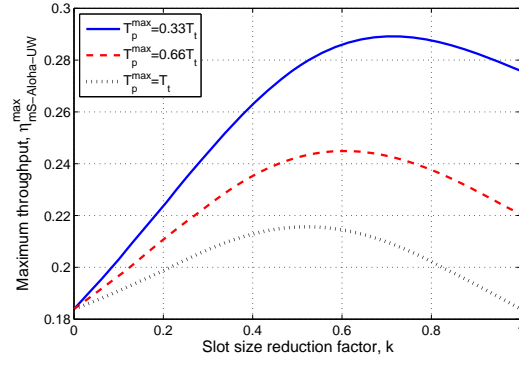
24

Figure 10: Throughput maximization via controlling k, with $T_p^{\max}$ as the parameter.

throughput gain is defined as:

$$Gain = \frac{\eta_{\text{mS-Aloha-uw}}^{\max} - \eta_{\text{S-Aloha-uw}}^{\max}}{\eta_{\text{S-Aloha-uw}}^{\max}} \times 100 \qquad (34)$$

Gain at a smaller values of $T_p^{\max}$ is less, which is because of a smaller system idling possibility with lesser $T_p^{\max}$, and hence the room for improved performance at an optimal k is also less.

The variation of maximum system throughput as a function of $\frac{T_p^{\max}}{T_t}$ (by controlling the nodal communication range R), with k as parameter, is shown in Fig. 11. The plots clearly indicate the
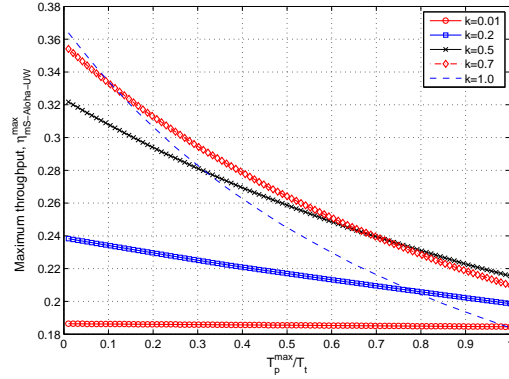

Figure 11: Variation of maximum system throughput as a function of $\frac{T_p^{\max}}{T_t}$.

importance of choosing right k for a given ratio $\frac{T_p^{\max}}{T_t}$, because no particular value of k offers the highest throughput performance as the propagation delay factor is increased.

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In Fig. 12, on the Y1 axis the optimum slot size reduction factor k that achieves $\eta^{\text{max}}_{\text{mS-Aloha-uw}}$ is plotted with respect to $\frac{T_p^{\text{max}}}{T_t}$, which can be controlled either by varying R or T_t. In conjunction,
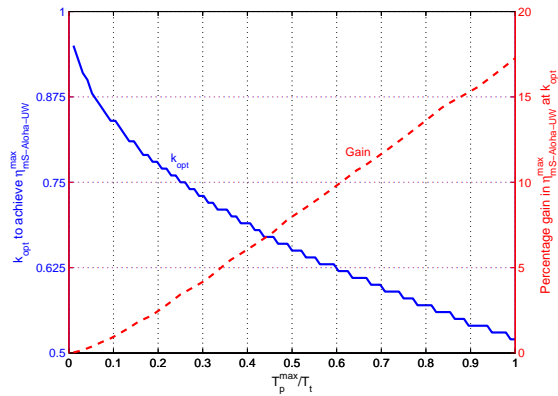

Figure 12: Optimum slot size reduction factor k_{opt} for maximum achievable throughput $\eta^{\text{max}}_{\text{mS-Aloha-uw}}$, and the corresponding maximum throughput gain with respect to naive S-Aloha-uw, as a function of propagation delay to transmission delay ratio.

the percentage throughput gain with respect to the naive S-Aloha-uw (defined in (34)) at the k_{opt} values is plotted on the Y2 axis. The plots further demonstrate that, while naive S-Aloha-uw does not offer a system throughput as good as in S-Aloha-rf, an optimal choice of slot size can offer an appreciable increase in throughput, especially for a large nodal coverage range.

7. Conclusion

In this paper, we have presented a theoretical framework for throughput performance computation of the basic random access protocols, namely Aloha and S-Aloha, in underwater wireless networks with a random internodal signal propagation delay. We have shown that, pure Aloha throughput performance does not have any impact, while S-Aloha does have a strong impact, of signal propagation speed. Further, we have proposed a new aggressive slotting concept, wherein the slot size can be optimally chosen such that, even by allowing some collisions due to overshooting the slot boundary, the overall system throughput can be significantly increased. The validity of our general analysis has been proven to hold for the special cases of conventional underwater

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slotted Aloha as well as in short-range RF propagation environments. Our analytic conclusions have been verified by discrete event simulations. The developed framework in the current study could be useful to benchmark the performance of advanced multiaccess protocols in propagation delay intensive ad hoc networks.

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