ELL 100 - Introduction to Electrical Engineering

Lecture 4: Circuit Analysis

Delta – Star Transformations
Outline

- Introduction
- Star Connection
- Delta Connection
- Delta to Star Transformation
- Star to Delta Transformation
- Equivalent Resistance of Circuit
- Exercise/Numerical Analysis
Star connections is generally used in long distance transmission lines as insulation requirement is less in star connection.
Delta connections are generally used in distribution networks for short distances.
Introduction

Alternators and generators are usually star connected.

600 MW Turbo-Generator at power plant
Transformer windings are connected in Star/Delta Connections.

A Three Phase Transformer with Name Plate
Generating transformer near to power plant generator are connected in star connection to provide grounding protection.
AC motors winding are connected in star/delta connection depending on requirement and application.
INTRODUCTION

Star and Delta connections are used in starting of three phase induction motors using STAR-DELTA Starter.

Star Delta Starter for Three phase Induction Motor
Delta-Star Starters are installed in cement industries for high inertial load applications.
INTRODUCTION

Power capacitors in 3 phase capacitor bank connections are either delta connected or star (wye) connected.

Delta connected capacitor bank

Delta connection of capacitors
The application of such connection is also used in high voltage direct current (HVDC) systems.
INTRODUCTION

The application of such connection is also used in Wheatstone bridge resistance measurement device.

Wheatstone Bridge
INTRODUCTION

STAR/Delta transformations and equivalent circuit calculations help in simplification and understanding of complex electrical circuits.

Complex Electric Circuits
Star/Delta connection is an arrangement of passive elements R, L and C such that the formed shape resembles a star or a delta symbol.

These connection are neither series and nor parallel.

Such connections are simplified using star-to-delta or delta-to-star conversion.

Such connections are found in complex DC circuits, full bridge rectifiers.

Such connections has larger application in three phase AC system.
A star network is rearranged form of Tee (T) network.
Three ends of resistors are connected in wye (Y) or star fashion. A common node point of star connection is known as neutral.
Three ways in which star connection may appear in a circuit.
When three resistors are connected in a fashion to form a closed mesh Δ, connection formed is known as Delta Connection.
DELTA CONNECTION

Three ways in which delta connection may appear in a circuit.
Three resistors $R_{AB}, R_{BC}$ and $R_{CA}$ connected in delta form and its equivalent star connection is shown below.

Delta and its equivalent Star
Δ ─────────────────── DELTA TO STAR TRANSFORMATION ─────────────────── Δ

- Two arrangements shown are electrically equivalent.
- Resistance between A and B for star = Resistance between A and B for delta.
- Therefore,

\[ R_A + R_B = R_{AB} \parallel (R_{BC} + R_{CA}) \]  \hspace{1cm} (1)

\[ R_A + R_B = \frac{R_{AB} (R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \]  \hspace{1cm} (2)
• Similarly for resistance between two terminals B-C and C-A,

\[ R_B + R_C = \frac{R_{BC} (R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \quad (3) \]

\[ R_C + R_A = \frac{R_{CA} (R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad (4) \]
The objective is to find $R_A$, $R_B$, and $R_C$ in terms of $R_{AB}$, $R_{BC}$, and $R_{CA}$.

Subtracting (3) from (2) and adding to (4) we obtain,

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$  \hspace{1cm} (5)$$

$$R_B = \frac{R_{BC} R_{AB}}{R_{AB} + R_{BC} + R_{CA}}$$  \hspace{1cm} (6)$$

$$R_C = \frac{R_{CA} R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$  \hspace{1cm} (7)$$
Delta to Star Transformation

- Easy way to remember delta to star transformation is,
  
  Any arm of star connection = \frac{\text{Product of two adjacent arms of } \Delta}{\text{Sum of arms of } \Delta}
Three resistors $R_A$, $R_B$ and $R_C$ connected in star formation and its equivalent delta connection is shown below.
**STAR TO DELTA TRANSFORMATION**

- Dividing (5) by (6) we obtain,
  \[
  \frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}} \tag{8}
  \]
  \[
  \Rightarrow R_{CA} = \frac{R_A R_{BC}}{R_B} \tag{9}
  \]

- Dividing (5) by (7) we obtain,
  \[
  \frac{R_A}{R_C} = \frac{R_{AB}}{R_{BC}} \tag{10}
  \]
  \[
  \Rightarrow R_{AB} = \frac{R_A R_{BC}}{R_C} \tag{11}
  \]
STAR TO DELTA TRANSFORMATION

• Substituting (9) and (11) into (5),

\[ R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \]  \hspace{1cm} (12)

• Similarly,

\[ R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \]  \hspace{1cm} (13)

\[ R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B} \]  \hspace{1cm} (14)
**Star To Delta Transformation**

- Easy way to remember star to delta transformation is, 
  Resistance between two terminals of $\Delta = $ 
  Sum of star resistances connected to those terminals + 
  product of same two resistances divided by the third
If a star network has all resistances equal to R, its equivalent delta has all resistances equal to ?

If a delta network has all resistances equal to R, its equivalent star has all resistances equal to ?
If a star network has all resistances equal to $R$, its equivalent delta has all resistances equal to $3R$.

If a delta network has all resistances equal to $R$, its equivalent star has all resistances equal to $R/3$. 
## SUMMARY

### Circuit

![Circuit Diagram]

### Conversion Formulas (Unbalanced)

<table>
<thead>
<tr>
<th>Formula</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_A = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$</td>
<td>When $Z_1 = Z_2 = Z_3 = Z$</td>
</tr>
<tr>
<td>$Z_B = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$</td>
<td>then $Z_A = Z_B = Z_C = \frac{Z}{3}$</td>
</tr>
<tr>
<td>$Z_C = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$</td>
<td></td>
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### Conversion Formulas (Balanced)

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<td>$Z_1 = \frac{Z_A + Z_C + Z_A Z_C}{Z_B}$</td>
<td>When $Z_A = Z_B = Z_C = Z_Y$</td>
</tr>
<tr>
<td>$Z_2 = \frac{Z_B + Z_C + Z_B Z_C}{Z_A}$</td>
<td>then $Z_1 = Z_2 = Z_3 = 3Z_Y$</td>
</tr>
<tr>
<td>$Z_3 = \frac{Z_A + Z_B + Z_A Z_B}{Z_C}$</td>
<td></td>
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</table>
The equivalent resistance of a circuit or network between its any two points (or terminals) is that single resistance which can replace the entire circuit between these points (or terminals).
DEFINITIONS

• STAR/DELTA CIRCUITS: These circuits generally possess star/delta configurations and needs to be simplified using necessary transformations and are converted into series parallel circuits.
**Equivalent Resistance of Star/Delta Circuit**

- The circuit is a combination of neither series nor parallel circuits.
**Equivalent Resistance of Star/Delta Circuit**

- **RULE:** Such circuit form star/delta. Use star delta transformation and convert to equivalent series-parallel circuit.
- Changing Delta formed by points A,B,C into equivalent Star,

\[
R_y = \frac{R \times R}{R + R + R} = \frac{R}{3}
\]
- The circuit formed is now combination of series-parallel circuit.
- The series-parallel circuit is further simplified to series circuit.
The circuit now becomes a simple series-parallel circuit and can be solved easily.
Q. Convert the Y network to an equivalent Δ network.
Soln:

\[ R_a = 7.5 + 5 + \frac{7.5 \times 5}{3} = 25 \text{ ohms} \]

\[ R_b = 7.5 + 3 + \frac{7.5 \times 3}{5} = 15 \text{ ohms} \]

\[ R_c = 5 + 3 + \frac{5 \times 3}{7.5} = 10 \text{ ohms} \]
Q. Convert the Δ network to an equivalent Y network.
EXERCISE/NUMERICAL ANALYSIS

Soln:

\[ R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = 5 \text{ ohms} \]

\[ R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5 \text{ ohms} \]

\[ R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 \text{ ohms} \]
Q. Using delta/star transformation, find equivalent resistance across AC.
Soln: Delta can be replaced by equivalent star-connected resistances,

\[ R_1 = \frac{R_{AB}R_{DA}}{R_{AB} + R_{DA} + R_{BD}} = \frac{10 \times 20}{10 + 40 + 20} = 2.86 \text{ ohms} \]

\[ R_2 = \frac{R_{AB}R_{BD}}{R_{AB} + R_{DA} + R_{BD}} = \frac{10 \times 40}{10 + 40 + 20} = 5.72 \text{ ohms} \]

\[ R_3 = \frac{R_{DA}R_{BD}}{R_{AB} + R_{DA} + R_{BD}} = \frac{10 \times 40}{10 + 40 + 20} = 11.4 \text{ ohms} \]
Figure now becomes,

\[ R_{AC} = 2.86 + \frac{(30 + 5.72)(15 + 11.4)}{(30 + 5.72) + (15 + 11.4)} = 18.04 \text{ ohms} \]
Q. Calculate equivalent resistance across terminals A and B.
Soln: Converting inner STAR (3 ohms, 3 ohms and 1 ohms) into Delta.

\[ R_1 = 3 + 3 + \frac{3 \times 3}{1} = 15 \text{ ohms} \]

\[ R_2 = 3 + 1 + \frac{3 \times 1}{3} = 5 \text{ ohms} \]

\[ R_3 = 1 + 3 + \frac{1 \times 3}{3} = 5 \text{ ohms} \]
Circuit now becomes,
Delta-connected resistances 1 Ω, 5 Ω and 8 are converted in star,

\[ R_1' = \frac{1 \times 8}{1 + 5 + 8} = \frac{4}{7} \text{ ohms} \]
\[ R_2' = \frac{5 \times 1}{1 + 5 + 8} = \frac{5}{14} \text{ ohms} \]
\[ R_3' = \frac{8 \times 5}{1 + 5 + 8} = \frac{20}{7} \text{ ohms} \]
Circuit now becomes,

\[ R_{AB} = \frac{4}{7} + \left[ \left( \frac{5}{14} + 2.5 \right) \left( \frac{20}{7} + \frac{20}{9} \right) \right] + 7.6 = 10 \text{ ohms} \]
Q. Calculate equivalent resistance across terminals A and B.
Soln: Replacing inner STAR into DELTA.
15.8 ohm is in parallel with 5 ohm and 26.3 ohm is in parallel with 4 ohm, circuit becomes
Converting upper delta into star,
Now equivalent resistance can be calculated as,

\[ R_{eq} = (3.8 + 2.98) \parallel (1.99 + 3.5) + 1.2 \]

\[ = 4.23 \text{ ohms} \]
Q. Obtain the equivalent resistance $R_{ab}$ for the circuit and use it to find current $i$. 
Soln: In this circuit, there are two Y networks and three Δ networks. Transforming just one of these will simplify the circuit.
We convert the Y-network comprising the 5-Ω, 10-Ω, and 20-Ω resistors into delta.

\[
R_1 = 5 + 10 + \frac{5 \times 10}{20} = 17.5 \text{ ohms}
\]

(comes in parallel with 12.5 Ω)

\[
R_2 = 5 + 20 + \frac{5 \times 20}{10} = 35 \text{ ohms}
\]

(comes in parallel with 15 Ω)

\[
R_3 = 10 + 20 + \frac{10 \times 20}{5} = 70 \text{ ohms}
\]

(comes in parallel with 30 Ω)
Combining the three pairs of resistors in parallel, we obtain.

\[ R_{ab} = \left( \frac{7.292 + 10.5}{21} \right) \Omega = \frac{17.792 \times 21}{17.792 + 21} = 9.632 \text{ ohms} \]

\[ i = \frac{V_s}{R_{ab}} = \frac{120}{9.632} = 12.458 A \]
Q. Determine the load current in branch EF in the circuit shown.
Sol. ACGA forms delta, Converting it to equivalent star.

\[ R_{AN} = \frac{200 \times 500}{900} = 111.11 \text{ohms} \]
\[ R_{GN} = \frac{500 \times 200}{900} = 111.11 \text{ohms} \]
\[ R_{CN} = \frac{200 \times 200}{900} = 44.44 \text{ohms} \]
Circuit can be redrawn as

\[ R_{NEF} = 111.11 + 600 = 711.11\, \text{ohms} \]
\[ R_{ND} = 600 + 44.44 = 644.44\, \text{ohms} \]
Branches NCD and NEF are in parallel, $711.11 \parallel 644.44 = 338$ ohms.

Total current $I$ in the circuit is calculated as:

$$I = \frac{V}{R_{eq}} = \frac{100}{111.11 + 338} = 0.222\,A$$
To obtain current in branch EF, we apply current division formula.

\[
I_{NEF} = I \times \frac{R_{NCD}}{R_{NCD} + R_{NEF}}
\]

\[
= 0.222 \times \frac{644.44}{711.11 + 644.44}
\]

\[
= 0.1055 A
\]
Q. A square and its diagonals are made of a uniform covered wire. The resistance of each side is 1 Ω and that of each diagonal is 1.414 Ω. Determine the resistance between two opposite corners of the square.
Q. Determine the resistance between the terminals A and B of the network.
Q. Find the current in 10 Ω resistor in the network shown by star-delta transformation.
Q. Using star/delta transformation, determine the value of R for the network shown such that 4Ω resistor consumes the maximum power.
REFERENCES

