ELL 100 - Introduction to Electrical Engineering

Lecture 6: Node Analysis of Circuits
Outline

- Introduction
- Node Analysis
- Super Node Concept
- Nodal Analysis vs Mesh Analysis
- Exercise/Numerical Analysis
INTRODUCTION

Applications of nodal circuit analysis:

Voltage summing using op-amps

- Used in DACs (Digital-to-Analog converters)
Applications of nodal circuit analysis:

Wheatstone Bridge circuits

- Used to measure unknown resistances.
- Mechanical and civil engineers measure resistances of strain gauges to find the stress and strain in machines and buildings.
What is Nodal Analysis?

- Nodal analysis is a method that provides a general procedure for analysing circuits using node voltages as the circuit variables.

- Also called Node-Voltage Method.
INTRODUCTION

Types of Nodes in Nodal Analysis:

- **Non-Reference Node:** Node which has a definite Node-Voltage.

- **Reference Node:** It is the node which acts as reference point to all the other node. Also called Datum Node/Ground.

Node 0 is Reference node

Nodes 1, 2 are Non-Reference node
**INTRODUCTION**

Reference node indication

- **Chassis ground**: used in devices where the case, enclosure, or chassis acts as reference point for all circuits.
- **Earth ground**: used when the potential of the earth is used as reference.

Common symbols for indicating reference node,
(a) common ground,
(b) (earth) ground,
(c) chassis ground.
INTRODUCTION

Steps to find Node Voltages:

1. Select a node as the reference (ground) node. Assign voltages $v_1$, $v_2$, $v_3$, \ldots, $v_{n-1}$ to the remaining $n-1$ nodes.
2. Apply KCL to each of the $n-1$ non-reference nodes.
3. Use Ohm’s law to express the branch currents in terms of node voltages.
4. Solve the resulting ‘$n-1$’ simultaneous equations to obtain the unknown node voltages.

For ‘$n$’ nodes there will be ‘$n-1$’ simultaneous equations to be solved to get all the node voltages.
Example circuits depicting Nodes:
**NODE VOLTAGE ANALYSIS**

- Dual of Mesh Analysis.
- Involves the application of KCL equations (KVL for mesh).
- One of the nodes is taken as reference or datum node (node with maximum number of branches connected preferred).
- The reference node is assumed to be at ground or zero potential.
- Potentials of all the other nodes are defined with respect to reference node.
- KCL equations are written for each node except for the reference node. On solving, node voltages are obtained.
**Super Node Analysis**

**Definition of Super Node:**
when a dependent or an independent voltage source is connected between two non-reference nodes, then these nodes can be combined to form a generalised node which is known as **Super Node**.

Super node can be regarded as a surface enclosing the voltage source and its two nodes.

Node 2 and 3 can be combined into a Super-Node.
Properties of super-node:

• A super-node cannot be assigned a single voltage value
• A super-node requires application of both KCL and KVL to solve it.
• Any element can be connected in parallel with the voltage source forming the super-node without affecting the analysis.
• A super-node satisfies the KCL as like a simple node.
ADVANTAGES OF NODAL ANALYSIS

Nodal Analysis vs Mesh Analysis?

• Nodal analysis can be applied to both planar and nonplanar circuits. But Mesh analysis can be applied to planar circuits only.

Planar circuit: a circuit that can be drawn on a plane with no crossed wires.

• The choice between Node Voltage analysis and Mesh Current analysis is usually unambiguous.

• To choose between methods, pick the one that involves solving the fewest equations.
**Nodal Analysis vs Mesh Analysis**

**Nodal Analysis**

- The number of voltage variables, and hence simultaneous equations to solve, equals the number of nodes minus one.
- Every voltage source connected to the reference node reduces the number of unknowns by one.
- Nodal analysis is thus best for circuits with voltage sources.

**Mesh Analysis**

- The number of current variables, and hence simultaneous equations to solve, equals the number of meshes.
- Every current source in a mesh reduces the number of unknowns by one.
- Mesh analysis is thus best for circuits with current sources.
Q1. Calculate Node Voltages in the given circuit.
**Numerical**

**Soln:** There are 3 nodes:
0: ground, 1 & 2: non-reference nodes.

**Step 1.** Node voltages \( v_1 \) and \( v_2 \) are assigned. Also direction of branch currents are marked.

**Step 2.** Apply KCL to nodes 1 and 2.
KCL at Node 1:

\[ i_1 = i_2 + i_3 \]

KCL at Node 2:

\[ i_2 + i_4 = i_1 + i_5 \]

**Step 3.** Apply Ohm’s Law to KCL equations.

Ohm’s Law to KCL equation at Node 1:

\[ i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \Rightarrow 3v_1 - v_2 = 20 \quad \text{...(1)} \]
CONT'D.

Now, Ohm’s Law to KCL equation at Node 2:

\[ i_2 + i_4 = i_1 + i_5 \Rightarrow \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2 - 0}{6} \]

\[ \Rightarrow -3v_1 + 5v_2 = 60 \quad ... (2) \]

On solving (1) and (2) we get,

\[ v_1 = 13.33 \text{ V} \text{ and } v_2 = 20 \text{ V} \]
Q2. Calculate Node Voltages and Branch Currents in the given circuit.
**Solution (Soln):** There are 2 nodes:
0: ground, 1: non-reference node

**Step 1.** Node voltage \(v_1\) is assigned. Also, the direction of branch currents are marked.

**Step 2.** Apply KCL to node 1
KCL at Node 1:

\[ i_1 + i_2 + i_3 = 0 \]

**Step 3.** Apply Ohm’s Law to KCL equations.

Ohm’s Law to KCL equation at Node 1:

\[
\Rightarrow \frac{v_1 - 0}{12} + \frac{v_1 - 60}{7} + \frac{v_1 - 0}{4} = 0 \\
\Rightarrow v_1 = 18V
\]
Now, from $v_1$ we can find the branch currents ($i_1$, $i_2$ and $i_3$)

\[
i_1 = \frac{v_1 - 0}{12} = \frac{18}{12} = 1.5 \text{A}
\]

\[
i_2 = \frac{v_1 - 60}{7} = \frac{18 - 60}{7} = -6 \text{A}
\]

\[
i_3 = \frac{v_1 - 0}{4} = \frac{18}{4} = 4.5 \text{A}
\]
Q3. Calculate Node Voltages in the given circuit.
**NUMERICAL**

**Soln:** Here 4V voltage source is connected between Node-1 and Node-2 and it forms a Super-node with a 2Ω resistor in parallel.

\[ v_2 - v_1 = 4V \] (always, whatever may be the value of resistor in parallel). So 2Ω resistor is irrelevant for applying KCL to the super-node.
KCL in terms of nodes voltages:

\[ 5 = \frac{v_1 - 0}{6} + \frac{v_2 - 0}{3} + 2 \]

\[ \Rightarrow 30 = v_1 + 2v_2 + 12 \]

\[ \Rightarrow v_1 + 2v_2 = 18 \quad (1) \]

\[ v_1 + 4 - v_2 = 0 \]

\[ \Rightarrow v_2 = v_1 + 4 \quad (2) \]

On solving (1) and (2) we get,

\[ v_1 = 3.33 \text{ V and } v_2 = 7.33 \text{ V} \]
Q4. Calculate current through the 4Ω resistor in the given circuit.
**Soln.** Here 6V voltage source is connected between Node-1 and Node-2 and it forms a Super node with a 4Ω resistor in series.

The two node voltages are related as:

\[ v_1 - v_2 = 6 \quad \text{...(1)} \]
KCL for this super-node 1 and 2:

\[ \frac{v_1 - 0}{3} + \frac{v_2 - v_3}{4} = 2 \]

\[ \Rightarrow 4v_1 + 3v_2 - 3v_3 = 24 \quad \text{...(2)} \]

And applying KCL at node 3:

\[ \frac{v_3}{5} + \frac{v_3 - v_2}{4} = -7 \]

\[ \Rightarrow 9v_3 - 5v_2 = -140 \quad \text{...(3)} \]
On solving (1), (2) and (3) we get,

\[ v_1 = -2.77 \text{ V}, \ v_2 = -8.77 \text{ V} \quad \text{and} \quad v_3 = -20.43 \text{ V} \]

Now, current through 4\(\Omega\) resistor is:

\[ \frac{v_2 - v_3}{4} = \frac{-8.77 - (-20.42)}{4} = 2.9125 \text{ A} \]
Q5. Solve for voltage $V$ across the $3\Omega$ resistor using node voltage analysis for the given circuit.
**NUMERICAL**

**Soln:** Here 6V voltage source is connected between Node-3 and Node-4, so node 3 and 4 are constrained to one another.

To find the number of independent nodes, the circuit is redrawn as shown below by turning off the sources.
There are three nodes, in which two of them are independent.

However, if we add the two series resistors (2Ω and 3Ω) then we will have only one independent node i.e. node 1 only.

And hence we will have to solve only one equation.

Then using voltage divider rule we can find the unknown voltage across 3Ω resistor.
Applying KCL at node 1,

\[ 4 = \frac{v_1 - 6}{1} + \frac{v_1 - 0}{5} + 5 \]

\[ \Rightarrow 6v_1 = 25 \]

\[ \Rightarrow v_1 = 4.1667\text{V} \]

Now, voltage across 3Ω resistor is

\[ v = 4.1667 \times \frac{3}{2 + 3} = 2.5\text{V} \]
Q6. Find $I_s$ for the circuit shown. Take $V_0 = 16V$. 
**Numerical**

**Soln.** Applying KCL at node 1:

\[ I_s = \frac{V_1}{6} + \frac{V_1 - V_0}{4} \]

\[ \Rightarrow I_s = \frac{5V_1}{12} - 4 \]  \hspace{1cm} (1)

KCL at node 2 gives:

\[ \frac{V_1}{4} = \frac{V_0 - V_1}{4} + \frac{V_0}{8} \]

\[ \Rightarrow \frac{2V_1}{4} = 8 \]

\[ \Rightarrow V_1 = 12\text{V} \]  \hspace{1cm} (2)

\[ I_s = \frac{5 \times 12}{12} - 4 = 5 - 4 = 1\text{A} \]
Q7. Solve the following network using the nodal analysis and determine the current through 2S resistor. (S indicates Siemens i.e. conductance)

\[
\begin{align*}
-3A \\
1 \quad 3S \\
2 \quad 2S \\
3 \quad v_3 \\
4S \\
5S \\
-8A \\
-25A \\
1S \\
0
\end{align*}
\]
**NUMERICAL**

**Soln:** Here there are 4 nodes, node 0 is reference node. So we have to write to 3 KCL equations.

\[-3A\]

![Diagram of a network with nodes labeled 1, 2, 3, and 4, and connections indicating currents and voltages.](image)
We can write nodal voltage equation in matrix form, as below

\[
\begin{bmatrix}
4 + 3 & -3 & -4 \\
-3 & 3 + 2 + 1 & -2 \\
-4 & -2 & 4 + 2 + 5
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}
= \begin{bmatrix}
-(3) + (-8) \\
-(-3) \\
-(-25)
\end{bmatrix}
\]

\[
\begin{bmatrix}
7 & -3 & -4 \\
-3 & 6 & -2 \\
-4 & -2 & 11
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}
= \begin{bmatrix}
-11 \\
3 \\
25
\end{bmatrix}
\]
On solving, we get

\[ v_1 = 1\text{V}, \quad v_2 = 2\text{V} \quad \text{and} \quad v_3 = 3\text{V} \]

Putting the value of \( v_2 \) to find current through 2S resistor we have

\[ I_{2S} = \frac{v_2 - v_3}{R} = G(v_2 - v_3) = 2(2 - 3) = -2\text{A} \]
NUMERICAL

Q8. Determine the voltages at the nodes of the given circuit.
**NUMERICAL**

**Soln:** Here there are 4 nodes, node-0 is reference node.

There are 3 non-reference nodes in this network. So we have to write three KCL equations.

Assign voltages to the three nodes as shown below and label the currents.
Applying KCL at node 1:
\[ 3 = i_1 + i_x \]

\[ \Rightarrow 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2} \]

\[ \Rightarrow 3v_1 - 2v_2 - v_3 = 12 \quad (1) \]

Applying KCL at node 2:
\[ i_x = i_2 + i_3 \]

\[ \Rightarrow \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4} \]

\[ \Rightarrow -4v_1 + 7v_2 - v_3 = 0 \quad (2) \]
Applying KCL at node 3:

\[ i_1 + i_2 = 2i_x \]

\[ \Rightarrow \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2} \]

\[ \Rightarrow 2v_1 - 3v_2 + v_3 = 0 \]  \hspace{1cm} (3)

On solving (1), (2) and (3) we get,

\[ v_1 = 4.8V, \ v_2 = 2.4V \text{ and } v_3 = -2.4V \]
Q1. Determine the node voltages and the current $i_x$ for the given circuit having 4 nodes with the following values, $R_1 = 2\Omega$, $R_2 = 1\Omega$, $R_3 = 5\Omega$, $R_4 = 6\Omega$ and $I_1 = 6A$. 
Q2. Determine the node voltages and the current through $8\Omega$ resistor for the given circuit having 4 nodes.
Q3. Determine the node voltages for the given circuit having 4 nodes.
Q4. Determine the node voltages and the current through $2\Omega$ Resistor, for the given circuit having 3 nodes.
Q5. Determine the node voltages using supernode analysis, for the given circuit having 5 nodes.
Q5. Determine the node voltages using supernode analysis, for the given circuit having 5 nodes.
Q6. Using Nodal analysis, calculate the node voltages $v_1$ and $v_2$ in the given circuit.
Q7. Using Nodal analysis, calculate the node voltages $v_1$ and $v_2$ in the given circuit and the current in the $8\Omega$ resistor.
Q8. Using Nodal analysis, calculate the node voltages $v_1$ and $v_2$ in the given circuit.
REFERENCES
